WATER TURBINES

Contributions to Their Study
Computation and Design

BY

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THE AMERICAN HIGH SPEED RUNNERS FOR WATER TURBINES

Looking at a modern American standard runner for a water turbine one is liable to wonder why the design of such runners is considered to be one of the most difficult problems in hydraulic engineering. The forms of the runners are so natural, the buckets and their curvature so simple, that "we fail to see where the pretended difficulties are." As is usual in such cases, we forget here again that there is always a direct proportion between the simplicity of a machine and the amount of brainwork and time necessary to produce the same. A brief history of the evolution of the American turbine or a glance at the reports of the numerous tests made in the Holyoke testing flume would convince us of this fact. Indeed, the American standard runners, as they are manufactured now, represent a great amount of hard and earnest work. Hundreds of tedious and expensive experiments, with many a failure and success, had to be made—an experience of almost half a century had to be aggregated first, before this modern runner type was produed.

The aim was, first, of course, a good efficiency. But this was not all. Already in the early eighties—at a time when the European engineers still were questioning the advantages of the radial inward flow turbine— there were in this country wheels of this type, which yielded efficiencies up to 84%, acording to the tests made in Holyoke. And yet since that time remarkable progress has been made. Following the general tendency of modern engineering, the speed and capacity of the turbines had to be steadily increased. That for the turbine designer this resulted in new difficulties is evident, as high speed calls for small dimensions, while high capacity calls for large dimensions, and consequently the increase of both is possible only to a certain limit.

The purpose of this study is to show how far the American manufacturers of water turbines have come in this respect and also to compare the results which were obtained by their various runner types. The accuracy of this study is naturally limited by the accuracy of the data, which were accessible to the writer, and which were taken from the guarantees of the different concerns. But as these guarantees are based on careful tests made in the Holyoke testing flume, and as these tests are considered official in this country, also the results of the following study can be considered reliable. The comparison at least will be accurate, because, if mistakes in the testing of the wheels be made (and some engineers, especially in Europe, believe that the Holyoke tests are not quite reliable regarding the actual discharge) the same mistakes owuld be made on all runners.

NOTATION.

To get a proper basis for this study, some of the principal turbine formulae must be recalled and some new ones derived. The notation is the same, which the writer uses in his lectures on water turbines at the University of Michigan.

$H\text{-}P$ = effective power of the runner.
N = speed of the runner in R. P. M.
Q = discharge of the runner in cubic feet per second.
H = net head in feet acting upon the turbine = gross head minus all losses in head race, conduit and tail-race.
ϵ = hydraulic efficiency of the turbine.
$(1-\epsilon)\,H$ = head lost inside of turbine itself due to friction, whirls and shocks.
D_1 = mean entrance diameter of runner in feet.
B = height of guide case in feet.
α_1 = angle between entrance speed and peripheal speed at D_1.
β_1 = bucket angle at D_1.
c_1 = real entrance speed at D_1.
w_1 = relative entrance speed at D_1.
v_1 = peripheral speed at D_1.
c_r = radial entrance speed at D_1 = radial component of c_1, see Figs. 1 and 2.

SPEED.

All modern American runners are of the radial inward flow type, working with pressurehead. The definition of the *"pressurehead turbine"* or *"pressure turbine"* (so-called reaction turbine) is: "The water enters the runner and flows through the same with a ecrtain pressurehead, as the whole available head is not turned into speed at the entrance. The real entrance speed

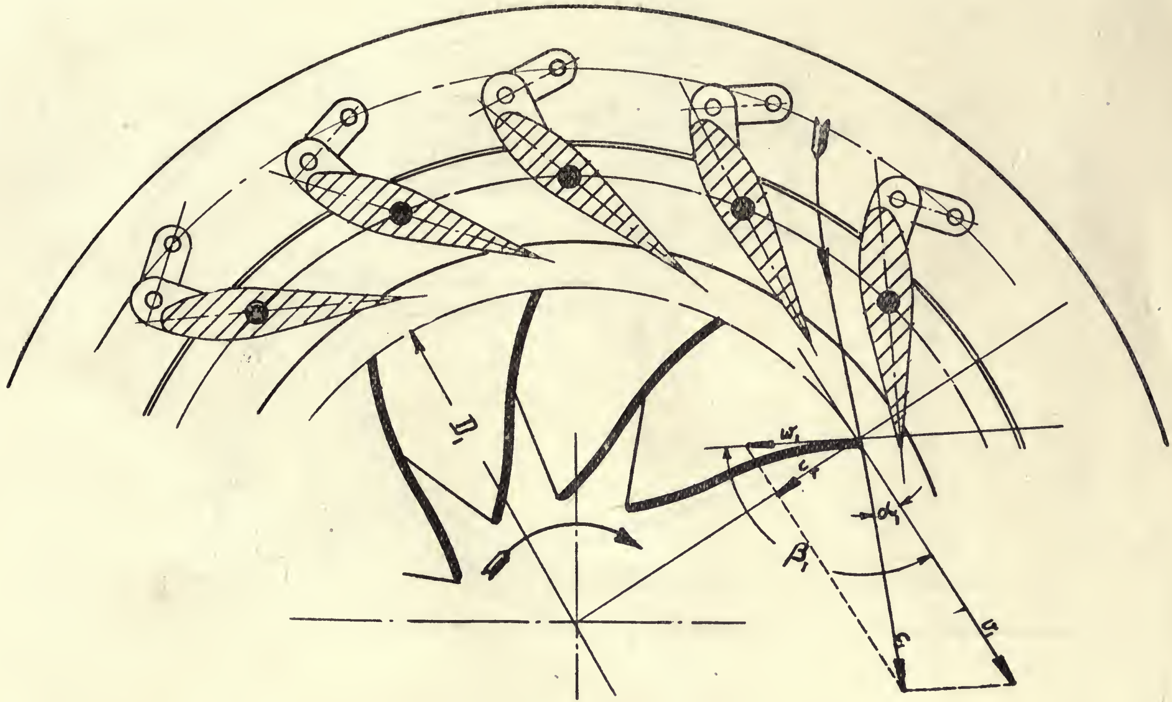


FIG. 1.

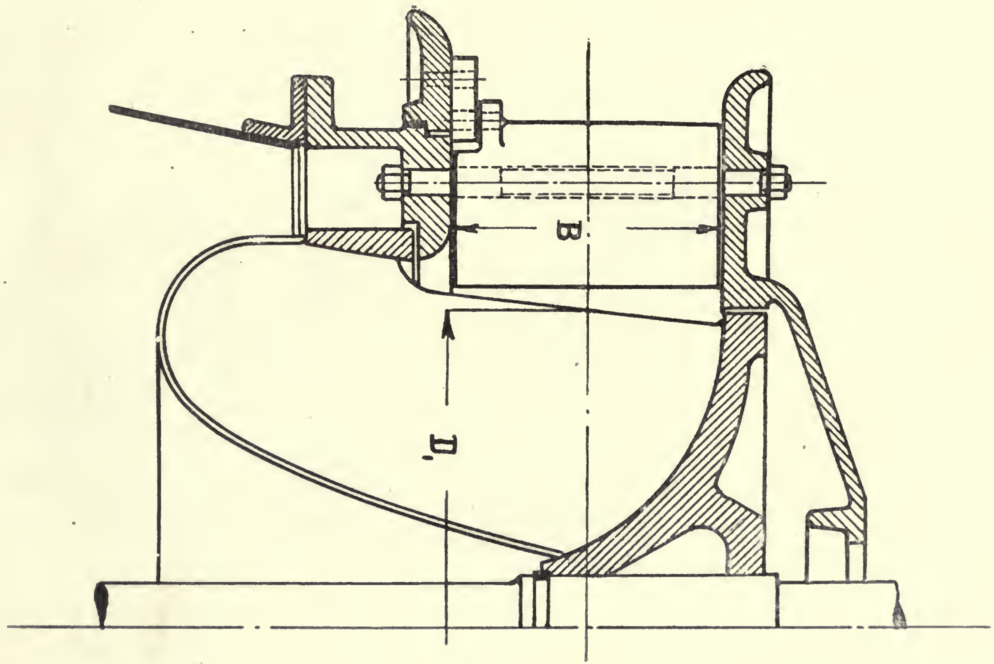


FIG. 2.

c_1 is smaller than the spouting velocity. A pressurehead is left, to be used for the acecleration of the flow of the water through the runner."

The regulation of the hydrodynamic conditions in the runner, for a flow either with or without pressurehead, is possible by the choice of the angles β_1 and α_1.

If the entrance into the runner is *"shockless,"* and the discharge is *"perpendicular,"* (real discharge speed perpendicular to the corresponding peripheral speed) then

$$c_1 = \sqrt{\epsilon g H} \sqrt{\frac{\sin \beta_1}{\sin(\beta_1 - \alpha_1) \cos \alpha_1}} \quad (1)$$

$$v_1 = \sqrt{\epsilon g H} \sqrt{\frac{\sin(\beta_1 - \alpha_1)}{\sin \beta_1 \cos \alpha_1}} \quad (2)$$

Both c_1 and v_1 are functions of the angles α_1 and β_1 for a given head. The speed c_1 can naturally never exceed the spouting veloicty $\sqrt{2g\epsilon H}$. It would become equal to this velocity if

$$\sqrt{\frac{\sin \beta_1}{\sin(\beta_1 - \alpha_1) \cos \alpha_1}} \sqrt{\epsilon g H} = \sqrt{2 g \epsilon H}$$

or if

$$\beta_1 = 2\alpha_1$$

For all angles β_1 which are larger than $2\alpha_1$, the speed c_1 will be smaller than the spouting velocity, hence the turbine will be a pressure turbine.

For a pressureless turbine the peripheral speed would be

$$v_1 = \frac{1}{2 \cos \alpha_1} \sqrt{2 g \epsilon H}$$

This is variable only within very small limits, as $\cos\alpha_1$ varies only a little for the values of α_1 which are used in practice. Hence the peripheral speed of the pressureless turbine is practically given by the head, and consequently the speed N (R.P.M.) can be varied only by variation of the runner diameter D_1. As practical reasons restrict both the increase and decrease of D_1, the speed of a pressureless turbine is variable only within narrow limits. This is one of the main reasons why nowdays pressure turbines occupy the first place, and pressureless turbines (Impulse wheels and Schwamkrug-turbines) are used only when absolutely necessary. The speed of the pressure turbine can be varied not only by variation of the runner diameter but also, and very effectively, by variation of the angles β_1 and α_1. Combining both means, it is easy to vary the speed of a pressure turbine for a given head and capacity in the ratio of 6 : 1.

To show how the angles α_1 and β_1 affect the peripheral speed v_1 the factor

$$\sqrt{\frac{\sin(\beta_1 - \alpha_1)}{\sin\beta_1 \cos\alpha_1}}$$

of equation (2) has been represented by a series of curves. Figure 3 gives the values of this factor for a series of constant bucket angles β_1 with variable angles α_1. Figure 4 gives the same values for a series of constant angles α_1 with variable angles β_1.

For $\beta_1 = 90°$ the factor

$$\sqrt{\frac{\sin(\beta_1 - \alpha_1)}{\sin\beta_1 \cos\alpha_1}} = 1$$

for all values of α_1. For all angles $\beta_1 < 90°$ the value of the radical is smaller than 1; for all angles $\beta_1 > 90°$ its value is larger than 1.

As a low or medium head turbine must, as a rule, be designed for a relatively high speed, all American standard runners, being built for low or medium heads, have $\beta_1 > 90°$ and are *"high speed runners."* Runners with $\beta_1 = 90°$ are called *"medium speed runners"* and those with $\beta_1 < 90°$ are called *"low speed runners."* See Figures 5, 6, 7.

Practical reasons, as for instance the necessity of an easy, smooth and yet a short curvature of the bucket, are limiting the increase of β_1. The value of $\beta_{1max} = 135°$ will represent good practice and will be found in many of the best American high speed runners. The increase of angle α_1 also increases the speed v_1 for all angles $\beta_1 > 90°$. But to avoid what is called *overgating,* it is advisable not to assume too high values for α_1. Tests show that the capacity of the runner reaches its maximum at a certain gate opening. To open more, is not only useless, but even

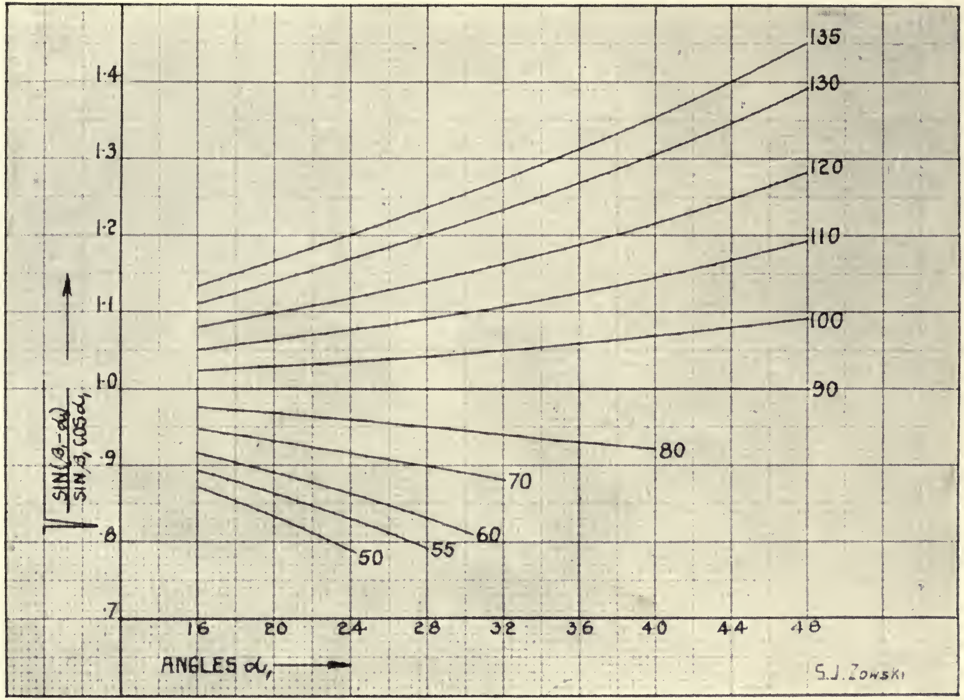
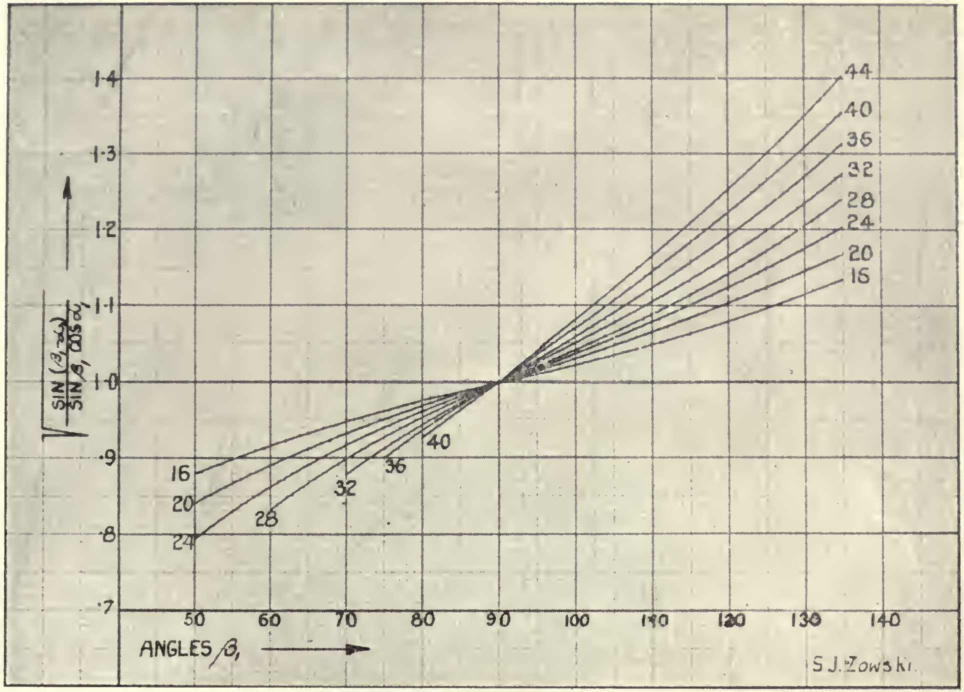

1·4
1·3
1·2
1·1
1·0
·9
·8
·7
$\sqrt{\frac{\sin(\beta_1-\alpha_1)}{\sin\beta_1 \cos\alpha_1}}$
135
130
120
110
100
90
80
70
60
55
50
16
20
24
28
32
36
40
44
48
ANGLES α_1
S.J. Zowski

Fig. 3.


1·4
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80
90
100
110
120
130
140
ANGLES β,
S J. Zowski.

FIG. 4.

wrong, as not only the output goes down, but also the efficiency. Although the point of overgating may, by a proper design of the runner, (at the least passage area) be moved upwards, it is not advisable to depend on this too much. Less efficiency than that expected does not disappoint the turbine buyer as much, as when the turbine is found to give less than the expected maximum power. It is not wise to make α_1 larger than 40°.

The hydraulic efficiency may be assumed between 0.82 and 0.84. For $\epsilon = 0.83$, $\sqrt{\epsilon g} = 5.167$ and

$$v_1 = 5.167 \sqrt{\frac{\sin(\beta_1 - \alpha_1)}{\sin \beta_1 \cos \alpha_1}} \sqrt{H}$$

Since for a given runner the value of the radical is a constant, we may write

$$v_1 = K_v \times \sqrt{H} \qquad (3)$$

and K_v may be called the *"speed constant."*

For $\beta_1 = 135°$, $\alpha_1 = 40°$, $\epsilon = 0.83$, $K_v =$ about 7.0. For given runners, where D_1, H and N are known, the speed constants may be calculated as follows:

$$K_v = \frac{v_1}{\sqrt{H}} = \frac{\pi D_1 N}{60 \sqrt{H}} \qquad (4)$$

and thus the different runner types may be compared in reference to speed. These constants will also show, whether a further increase of the speed is possible or not. Should the speed constant be considerably larger than 7, then it can be assumed with certainty that either the guaranteed speed is higher than the *best speed* (the *best speed* is the speed at which the runner yields the maximum efficiency) or that the nominal diameter of the runner is larger than the mean diameter D_1.

CAPACITY.

The quantity of water discharging from a given opening in a certain time, say in one sec., is $Q = const. \times \sqrt{H}$, if H is the head at the center of the opening. Hence, for a given opening,

$Q/\sqrt{H}$ = constant. We call this constant the *"specific discharge"* and use for the same the symbol:

$$Q_1 = \frac{Q}{\sqrt{H}} \text{ cub. ft./sec.} \tag{5}$$

The specific discharge from an orifice is the discharge in cub. feet per sec. when $H = 1$ ft.

Take into consideration the entrance area of the runner,

$$A_1 = \pi D_1 B \times k_1,$$

where k_1, being smaller than 1, is a factor, the addition of which is necessary, in order to consider the decrease of the circumference by the ends of vanes and buckets.

The speed of the stream, normal to this entrance area, is the radial speed c_r, which, like all speeds of a given runner—is in direct proportion to $\sqrt{H}$.

$$c_r = k_3 \sqrt{H}$$

Passage area × speed of flow normal to the area = discharge. Therefore

$$Q = A_1 c_r = \pi D_1 B k_1 k_3 \sqrt{H}.$$

Express the width of the guide case in parts of runner diameter D_1

$$B = k_2 D_1$$

(In runners of the same type k_2 will be nearly the same for all runner sizes.)

Then by substitution we obtain

$$Q = \pi D_1^2 k_2 k_3 \sqrt{H} = K_q D_1^2 \sqrt{H}, \tag{6}$$

where $K_q = \pi k_1 k_2 k_3$.

$$K_q = \frac{Q}{D_1^2 \sqrt{H}} = \frac{Q_1}{D_1^2} \tag{7}$$

K_q is the *"capacity constant"* of the runner. *The capacity constant of a runner is its specific discharge for a runner diameter = 1 ft.*

In all runners of the same type K_q will be nearly the same, hence the capacity constant is a criterion of the capacity of different runner types.

Fig. 5.
LOW SPEED RUNNER.

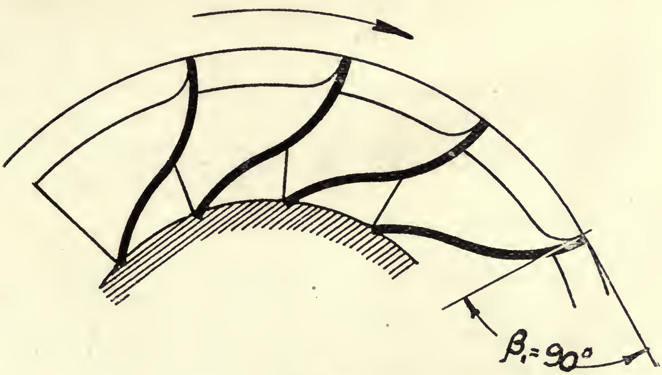

Fig. 6.
MEDIUM SPEED RUNNER.

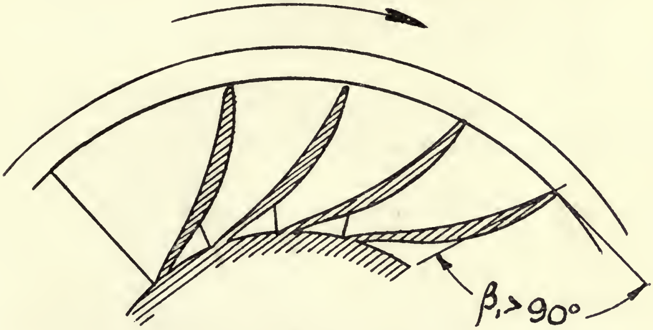

Fig. 7.
HIGH SPEED RUNNER.

SPEED AND CAPACITY.

Knowing the speed and capacity constants of different runners and runner types, we are not yet able to say to what extent they fulfill the requirements of *highest speed with highest capacity.*

We may have two runners with two different values of K_q and K_v, and yet both runners may be equivalent, when we consider capacity and speed *together.* Another criterion must be introduced, which will be a proper combination of K_q and K_v. This combination could be made in various ways, but the most convenient one was indicated by M. Baashuus and Professor Camerer and may be derived as follows:

$$v_1 = \frac{\pi D_1 N}{60} = K_v \sqrt{H}$$

$$N = \frac{60 \ K_v \sqrt{H}}{\pi D_1}$$

$$K_q D_1^2 = Q_1, \therefore D_1 = \sqrt{\frac{1}{K_q}} \sqrt{Q_1}$$

By substitution we obtain

$$N = \frac{60 K_v \sqrt{H}}{\pi \sqrt{\frac{1}{K_q}} \times \sqrt{Q_1}} = \frac{60 \sqrt{K_q} \times K_v}{\pi} \times \sqrt{\frac{H}{Q_1}}$$

The power of a turbine is

$$H\text{-}P = \frac{\gamma Q H}{550} \eta;$$

η = efficiency of turbine. $H\text{-}P = KQH$. As a rule η is taken $= 80\%$, then $K = 1/11$.

$$Q = H\text{-}P/KH, \text{ and } Q_1 = H\text{-}P/KH \sqrt{H}$$

Substitute in the last equation for N, then

$$N = \frac{60 \sqrt{K_q} \ K_v}{\pi} \times \frac{\sqrt{H}}{\sqrt{\frac{H\text{-}P}{K H \sqrt{H}}}}$$

$$N = \frac{60 \sqrt{K_q} \ K_v \sqrt{K}}{\pi} \times \frac{H \sqrt[4]{H}}{\sqrt{H\text{-}P}}$$

$$N = K_t \times \frac{H \sqrt[4]{H}}{\sqrt{H\text{-}P}} \qquad (8)$$

whereby

$$K_t = \frac{60 \sqrt{K_q}\, K_v \sqrt{K}}{\pi} \qquad (9)$$

K_t may be called the *"type constant"* or *"type characteristic"* of the runner. It is a combination of the speed and capacity constant, and both determine the type of the runner. The convenience of this constant will be apparent, when we write equation 8 in the following form:

$$K_t = \frac{N \sqrt{H\text{-}P}}{H \sqrt[4]{H}} \qquad (10)$$

K_t can be figured directly from the speed, power and head, which data can be obtained easily and seldom differ from actual values. No dimensions of the runner, neither the discharge nor the efficiency need to be known, and yet the efficiency is considered, because the formula

$$K_t = \frac{60 \sqrt{K_q} \times K_v}{\pi} \times \sqrt{K}$$

contains K which is a function of η.

Turbines of the same capacity and speed constant, but with different efficiencies, will have a different type characteristic. Hence K_t is *an absolute criterion for turbines in reference to the aim, "highest speed and highest capacity with best efficiency."* The meaning of K_t can be found by assuming $H\text{-}P = 1$ and $H = 1$, then $K_t = N$ (R.P.M.).

"The type characteristic of a runner is the speed in R.P.M. which would be attained by the runner, if it were reduced in all dimensions to such an extent, as to develop 1 H-P when working under the head $H = 1$ ft."

In Germany the term "specific speed" (spezifische Geschwindigkeit or spezifische Umlaufzahl) and the symbol N_s or n_s is used for K_t.

The writer prefers, however, not to use this term for the following reasons. The word *specific* discharge is used for $Q_1 =$

$Q/\sqrt{H}$, the word *specific* power is used for $H\text{-}P/H\sqrt{H}$, or for the discharge and power under the head $H=1$ ft. Hence *specific speed* should denote the speed at $H=1$ ft.

$$N_1 = \frac{N}{\sqrt{H}}$$

As N_1 is used very frequently, the term *type characteristic* has been chosen for K_t.

SPEED, CAPACITY AND TYPE CHARACTERISTICS OF THE AMERICAN HIGH SPEED RUNNERS.

I. THE DAYTON GLOBE IRON WORKS CO., DAYTON, OHIO.

The Dayton Globe Iron Works Co. is the manufacturer of the world known *American* turbines. The last two types developed by this concern are the *New American* and the *Improved New American turbine*. The difference in the design of these two runners is seen from figures No. 8, 9, 10, 11. In order to increase the capacity, the height has been increased, and the entrance edge inclined. Thus the mean diameter D_1 has been reduced and speed increased, while the minimum passage area at "*a*" is kept ample. The discharge end of the bucket has also been changed. In order to increase the actual discharge area, and to so decrease the discharge speed, the bucket has been drawn down and shaped spoonlike at the discharge. The radius of the curvature of the spoon at "*b*" however seems to be rather small. The outward discharge could be made more effective by a larger spoon. Nevertheless both the capacity and the efficiency of this runner are very good.

The data for a 19″ New American runner are:

$H=25$ ft. $H\text{-}P=80$. $N=339$.
60 $Q=2128$ cub. ft. per min.
(*a*) Speed constant.

$$D_1^{\text{Ft.}} = \frac{19}{12};\; K_v = \frac{\pi D_1 N}{60\sqrt{H}} = \frac{\pi \frac{19}{12} 339}{60\sqrt{25}}$$

$K_v = 5.62$.

Some engineers prefer to express all speeds in parts of the spouting velocity $\sqrt{2gH}$.

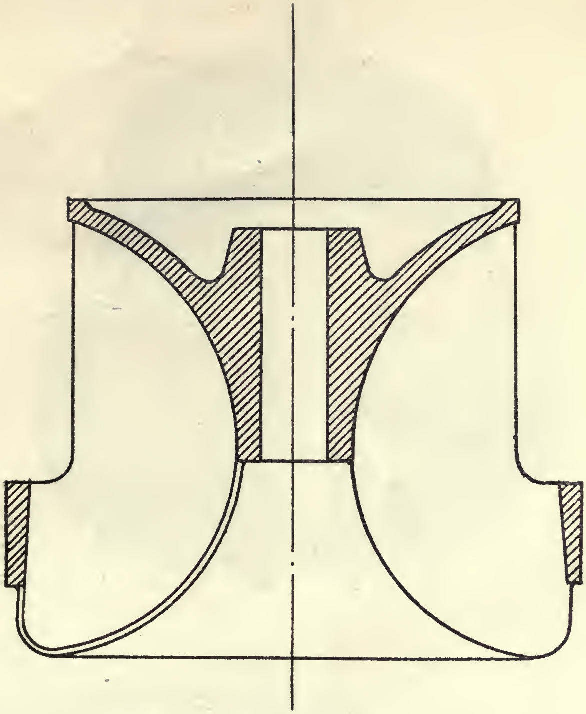

FIG. 8.
SECTION THROUGH NEW AMERICAN RUNNER.

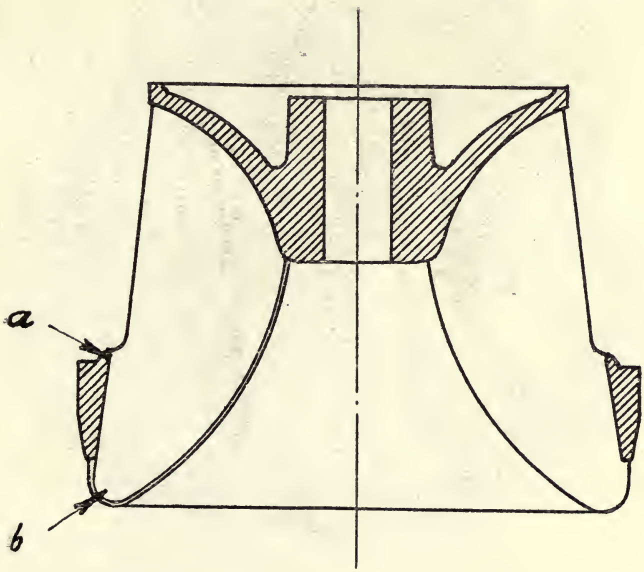

FIG. 9.
SECTION THROUGH IMPROVED NEW AMERICAN RUNNER.

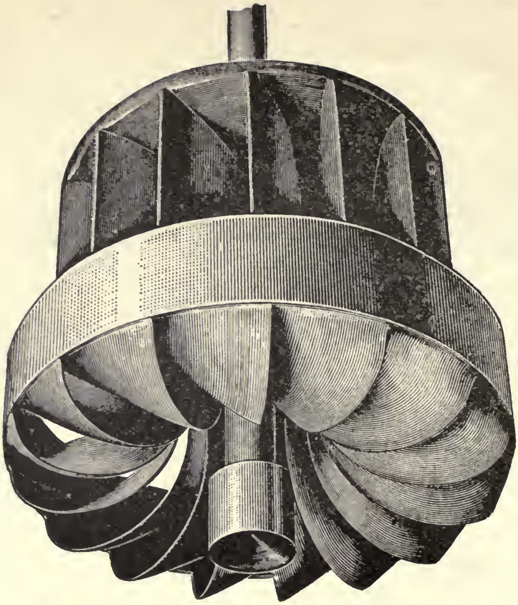

FIG. 10.
NEW AMERICAN RUNNER.

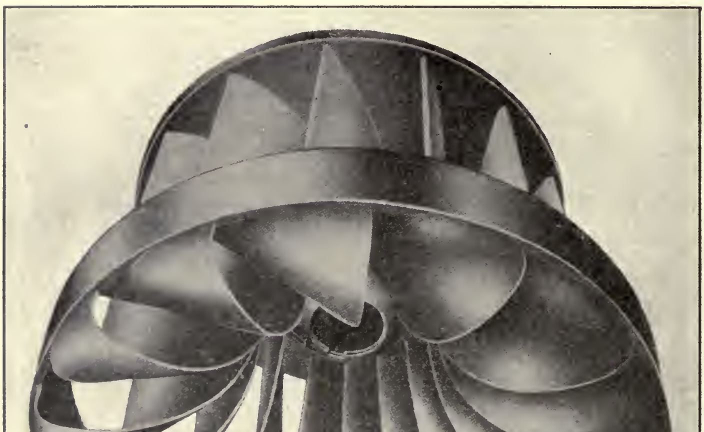

FIG. 11.
IMPROVED NEW AMERICAN RUNNER.

$$v_1 = K'_v \sqrt{2gH};\ K'_v = \sqrt{\frac{1}{2g}}\ K_v$$

$K_v =$ about 0.7.

(*b*) Capacity constant.

$$\text{Specific discharge } Q_1 = \frac{Q}{\sqrt{H}} = \frac{2128}{60\sqrt{25}}$$

$Q_1 = 7.0933.$

$$K_q = \frac{Q_1}{D_1^2} = \frac{7.0933}{\left(\frac{19}{12}\right)^2} = 2.829$$

(*c*) Type characteristic.

$$K_t = \frac{N\sqrt{H\text{-}P}}{H\sqrt[4]{H}} = \frac{339\sqrt{80}}{25\sqrt[4]{25}}$$

TABLE NO. 1.

NEW AMERICAN RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
10	1.95	0.176	122.4	5.35	2.81	51.3	0.667
13	2.96	0.264	99.0	5.61	2.52	50.9	0.700
16	4.80	0.432	80.4	5.60	2.70	52.8	0.699
19	7.09	0.640	67.8	5.61	2.83	54.2	0.700
22	9.28	0.840	58.4	5.60	2.76	53.5	0.699
25	11.63	1.048	51.4	5.60	2.68	52.7	0.699
27.5	15.04	1.360	46.6	5.60	2.87	54.4	0.669
30	18.20	1.648	42.8	5.60	2.90	55.0	0.699
33	21.60	1.984	39.2	5.63	2.85	55.3	0.703
36	27.50	2.488	35.8	5.62	3.05	56.5	0.702
39	29.83	2.704	32.8	5.59	2.82	54.0	0.697
42	33.36	3.024	30.6	5.60	2.72	53.2	0.699
45	40.05	3.624	28.6	5.60	2.84	54.5	0.699
48	46.08	4.080	26.8	5.61	2.88	54.2	0.700
51	49.27	4.464	25.4	5.65	2.72	53.7	0.705
54	57.93	5.256	23.8	5.61	2.85	54.5	0.700
57	63.39	5.744	22.6	5.61	2.81	54.3	0.700
60	73.73	6.680	22.0	5.75	2.85	56.8	0.717

In the same way the constants for the other runner sizes have been calculated and are given in Table No. 1. Also the specific speed $N_1 = N/\sqrt{H}$ and the specific power

$$H\text{-}P_1 = \frac{H\text{-}P}{H\sqrt{H}}$$

have been added, as both are very convenient characteristics of a runner. They may be used to calculate the speed and power of each runner for any given head.

$$N_1 = 339/\sqrt{25} = 67.8.$$
$$H\text{-}P_1 = 80/25\sqrt{25} = 0.64.$$

[At $H = 36$ this runner would have a speed $N = 67.8 \times \sqrt{36} = 406.8$ R.P.M. and would develop $0.64 \times 36\sqrt{36} = 142.2$ *H-P.*]

The Improved New American runner, figured in the same way, will be found to have a speed constant which is considerably larger than that corresponding to $\beta_1 = 135°$ and $\alpha_1 = 40°$. As the speed N (R.P.M.) must be assumed to be correct, (is based on Holyoke tests) and the angles β_1 and α_1 do not exceed 135° and 40° respectively, (β_1 is about 135°; α_1 seldom exceeds 35°), the discrepancy can be due only to the fact that the nominal diameter is larger than the mean diameter D_1. Measuring several runners, the writer has found that the nominal diameter is taken close to the fillet (see Fig. No. 9). The ratio of the mean to the nominal diameter was found to be about 0.97, for smaller runner sizes. For larger szes this ratio is somewhat smaller.

TABLE NO. II.

IMPROVED NEW AMERICAN RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
16	6.76	0.616	102.0	6.88	3.80	80.1	0.858
19	8.82	0.808	87.0	6.96	3.52	78.3	0.869
22	11.63	1.064	75.0	6.94	3.47	77.5	0.866
25	14.87	1.360	66.8	7.04	3.56	77.9	0.878
29	19.74	1.808	59.0	7.18	3.38	79.4	0.896
34	26.83	2.464	49.8	7.13	3.35	78.3	0.890
39	35.21	3.232	43.6	7.15	3.33	78.5	0.892
44	44.63	4.104	38.8	7.18	3.32	78.7	0.896
49	55.21	5.072	34.8	7.18	3.32	78.4	0.896
54	69.08	6.344	31.6	7.18	3.41	79.5	0.896
60	84.66	7.784	29.0	7.30	3.39	81.0	0.911
66	101.80	9.344	26.2	7.26	3.35	80.2	0.905

Table No. II has been calculated with $D_1 = 0.97 \times$ nominal diameter for all sizes.

The specific power and speed have been represented by curves, Fig. 12, and they show clearly that the success of the Improved

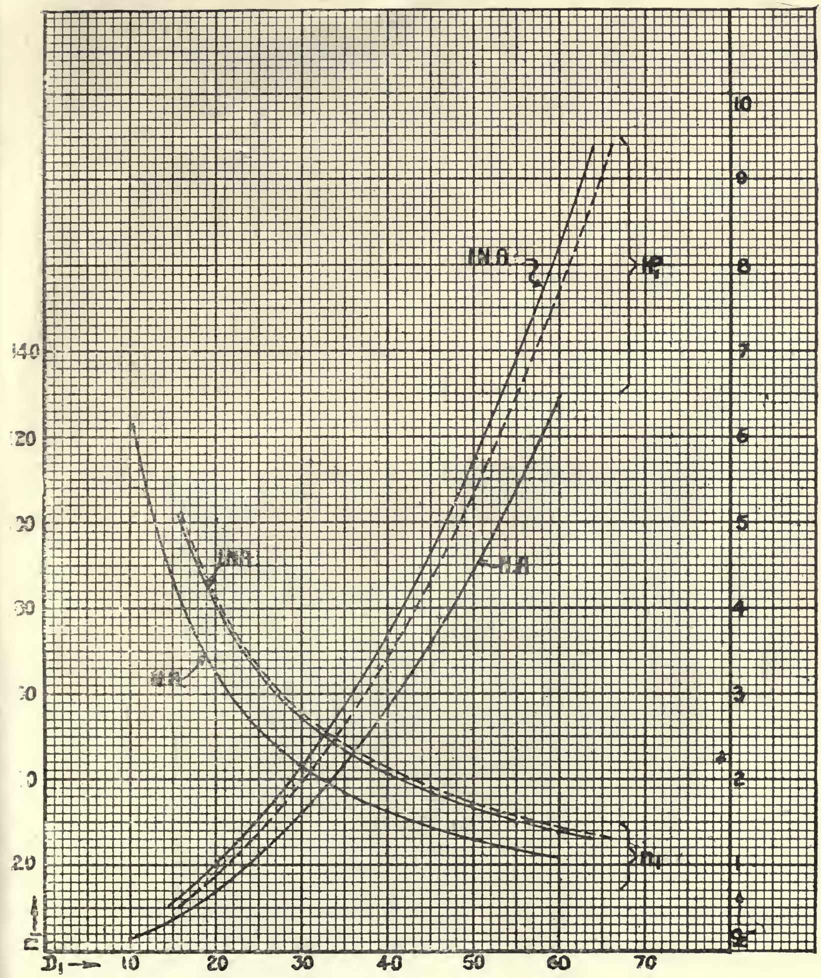

Fig. 12.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY THE DAYTON GLOBE IRON WORKS CO.

N. A.—NEW AMERICAN RUNNER.

I. N. A.—IMPROVED NEW AMERICAN RUNNER.

New American over the New American runner, regarding the aim of highest capacity and highest speed, is remarkable. The dotted curves give the same values, if the nominal diameters are taken as a basis.

The average values of the characteristic constants are:

	New American	Improved New American
Capacity const. K_q	2.8	3.43
Speed const. K_v	5.6	7.1
Type characteristic K_t	54.1	79.0

2. THE PLATT IRON WORKS CO., DAYTON, O., SUCCESSORS TO STILLWELL BIERCE CO.

To meet the demand of turbines for low, medium, or high heads, this company is manufacturing both radial inward and outward flow, pressure and pressureless tubines. Here only the

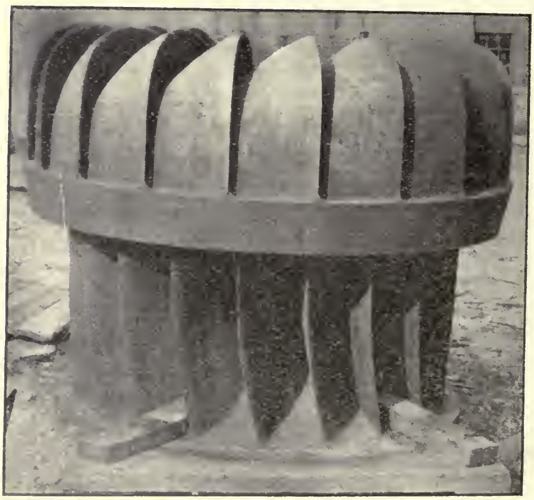

FIG. 13.
VICTOR RUNNER, TYPE A, INCREASED CAPACITY.

Victor turbine Type A for low and medium heads will be taken into consideration. The name under which this turbine generally appears is *Cylinder Gate Victor Turbine,* as this concern prefers

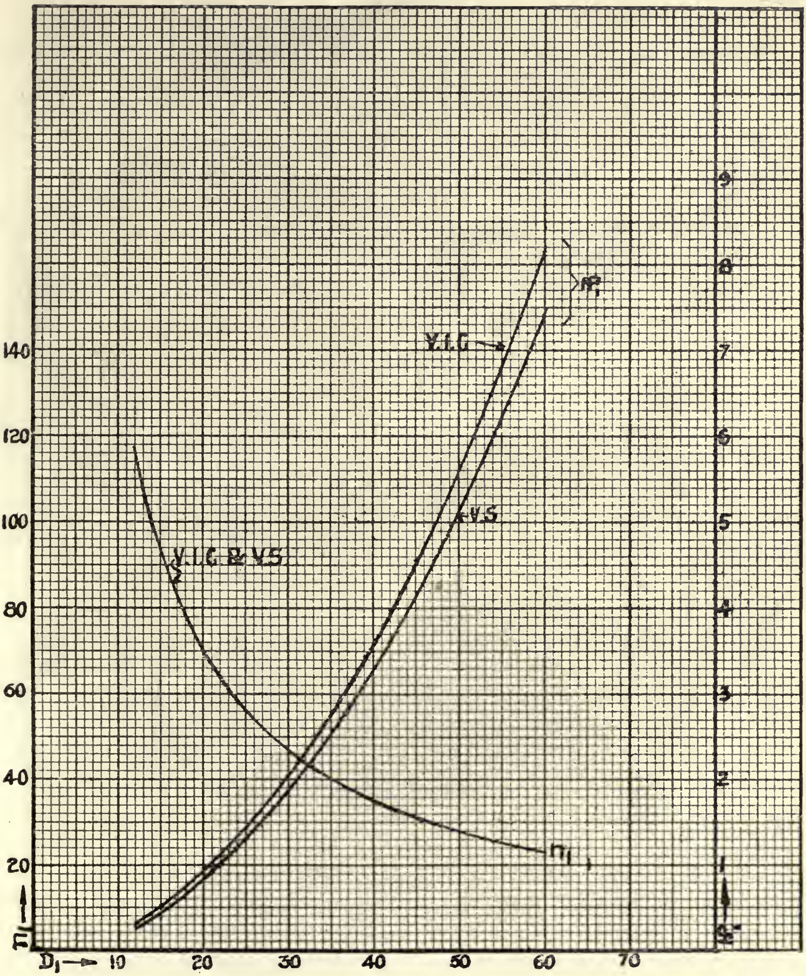

FIG. 14.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY THE PLATT IRON WORKS CO.

V. S.—VICTOR STANDARD CAPACITY RUNNER.

V. I. C.—VICTOR INCREASED CAPACITY RUNNER.

to equip its turbines with the cylinder gate regulating device. There are two patterns of the Victor runner Type A: the *Standard Capacity* and the *Increased Capacity runner.* Both have the same speed, but the capacity is different.

One of the characteristic features of the Victor runner is its large number of buckets. It is the opinion of the writer, however, that there are no reasons to use so many buckets, either for strength or for efficiency. On the contrary, it is advisable to reduce the number of buckets of low head runners, in order to increase the capacity and avoid small widths of the chutes at the runner hub. Tables III and IV have been calculated in the same way as the preceding tables.

TABLE NO. III.

VICTOR RUNNER, TYPE A, STANDARD CAPACITY.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
12	3.26	0.296	117.4	6.13	3.26	63.8	0.765
15	5.10	0.462	93.8	6.13	3.25	63.7	0.765
18	7.34	0.666	78.6	6.18	3.27	64.2	0.771
21	9.99	0.906	67.2	6.15	3.27	64.0	0.767
24	13.04	1.183	58.6	6.12	3.27	63.7	0.763
27	16.52	1.498	52.0	6.11	3.27	63.7	0.762
30	20.39	1.849	47.0	6.14	3.26	64.0	0.766
33	24.67	2.237	42.8	6.16	3.26	64.1	0.768
36	29.36	2.662	39.0	6.12	3.26	63.6	0.763
39	34.46	3.124	36.0	6.12	3.25	63.7	0.763
42	39.97	3.624	33.6	6.15	3.27	64.0	0.767
45	45.88	4.160	31.2	6.10	3.26	63.7	0.761
48	52.20	4.741	29.0	6.06	3.25	63.2	0.756
51	58.93	5.341	27.0	6.00	3.25	62.5	0.749
54	66.07	5.990	25.6	6.01	3.26	62.7	0.750
57	73.61	6.673	24.4	6.07	3.26	63.2	0.757
60	81.56	7.395	23.0	6.02	3.26	62.7	0.751

TABLE NO. IV.

VICTOR RUNNER, TYPE A, INCREASED CAPACITY.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
12	3.59	0.325	117.4	6.13	3.59	67.0	0.765
15	5.61	0.508	93.8	6.13	3.60	66.8	0.765
18	8.07	0.732	78.6	6.18	3.60	67.2	0.771
21	10.99	0.996	67.2	6.15	3.60	67.0	0.767
24	14.35	1.292	58.6	6.12	3.59	66.6	0.763
27	18.17	1.647	52.0	6.11	3.58	66.6	0.762
30	22.43	2.036	47.0	6.14	3.58	67.1	0.766
33	27.14	2.461	42.8	6.16	3.59	67.2	0.768
36	32.30	2.928	39.0	6.12	3.59	66.8	0.763
39	37.91	3.436	36.0	6.12	3.59	66.8	0.763

TABLE NO. IV.—*Continued.*

VICTOR RUNNER, TYPE A, INCREASED CAPACITY.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
42	43.96	3.986	33.6	6.15	3.59	67.2	0.767
45	50.46	4.576	31.2	6.10	3.58	66.7	0.761
48	57.42	5.206	29.0	6.06	3.59	66.2	0.756
51	64.82	5.877	27.0	6.00	3.57	65.5	0.749
54	72.68	6.590	25.6	6.01	3.59	65.7	0.750
57	80.97	7.342	24.4	6.07	3.58	66.2	0.757
60	89.72	8.135	23.0	6.02	3.59	65.6	0.751

The average values of the characteristic constants are:

	Victor Standard Capacity	Victor Increased Capacity
Capacity const. K_q	3.26	3.59
Speed const. K_v	6.1	6.1
Type characteristic K_t	63.5	66.6

THE JAMES LEFFEL AND CO., SPRINGFIELD, O.

The James Leffel & Co. manufacture the well known Double wheel, designed originally by James Leffel as a combination of two runners, one being a pure radial, the other a radial and downward discharge runner. To increase the capacity this runner had to be bulged out more, and so the new Double wheel was brought out, which, like all high speed runners, discharges both in central and outward direction. The special feature of this *Improved Samson Wheel* is the partition wall, subdividing the runner into two sections. The upper half is a solid casting, the lower half has steel plate buckets.

Although manufacturing reasons—such as the wish to use some existing patterns or pattern parts—may have been prevailing, it is more than doubtful whether the addition of the partition wall is an advantage. Without going any further into this matter, only a few reasons for this opinion of the writer shall be stated.

The partition wall increases the friction loss and decreases the effective height of the runner and thus its capacity. Further, it increases the possibility of clogging, and if not built so that it coincides with the corresponding water flow lines, it will decrease the capacity still more.

One advantage could be claimed, namely, that the regulation by a cylinder gate will not affect the efficiency of the turbine very much. But this would be true only for a small variation of load, when the cylinder gate closes only the upper part of the runner.

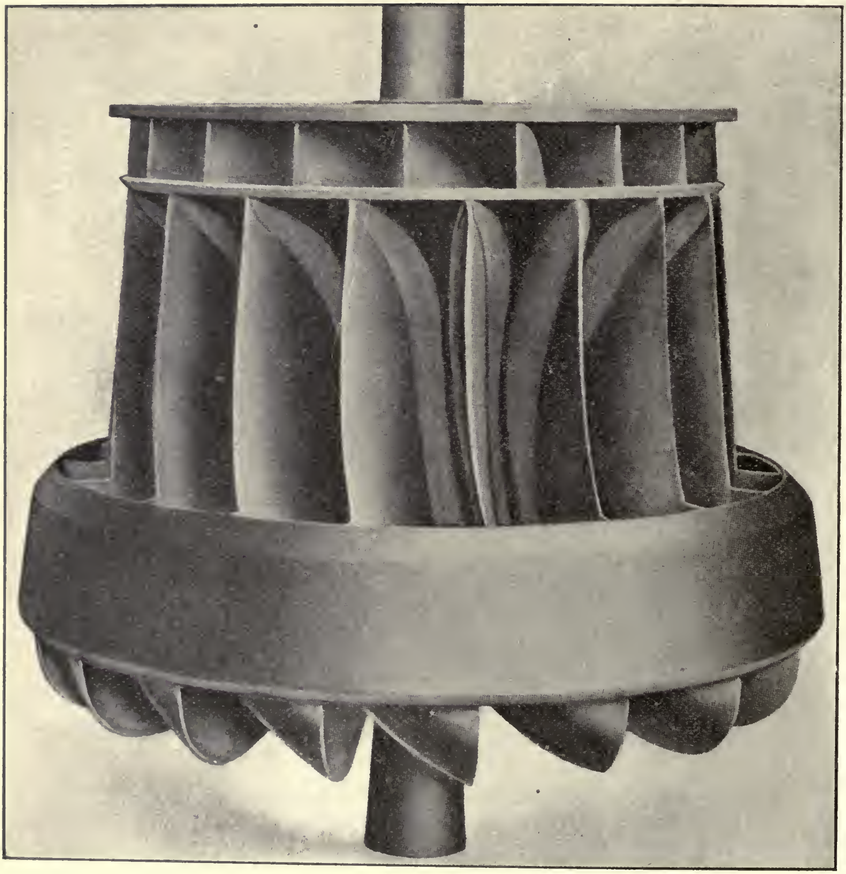

Fig. 15.
IMPROVED SAMSON RUNNER.

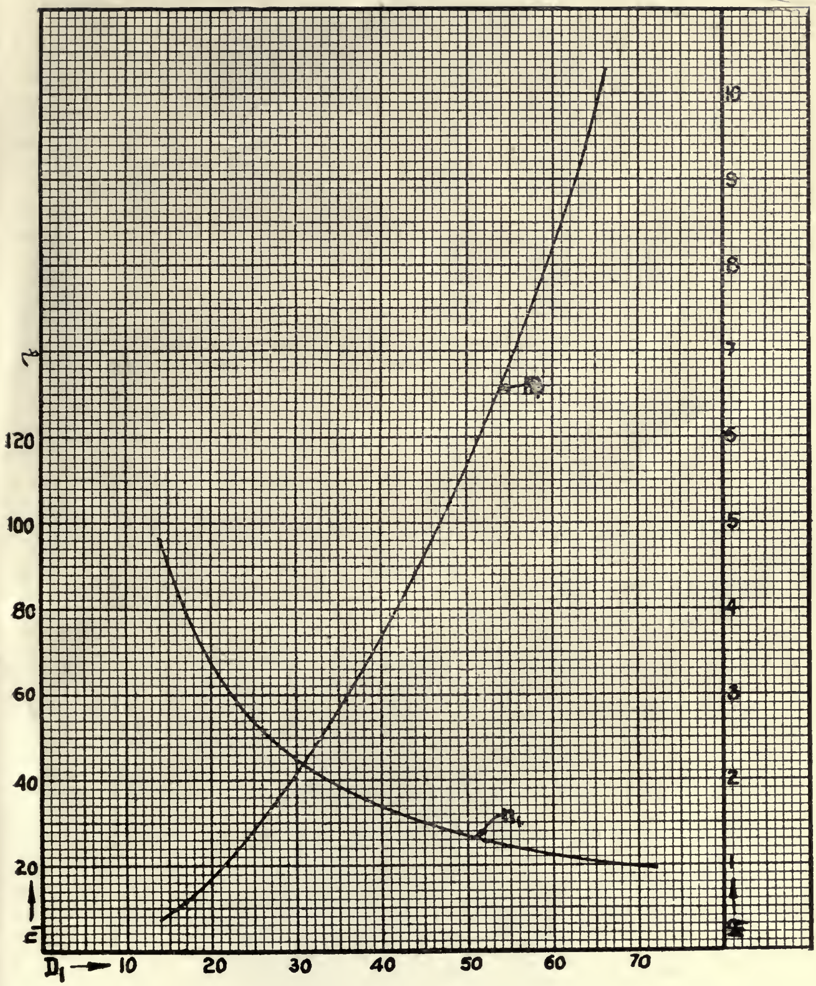

FIG. 16.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY JAMES LEFFEL & CO.

IMPROVED SAMSON RUNNER.

TABLE NO. V.

IMPROVED SAMSON RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
17	6.71	0.616	92.8	6.86	3.34	72.9	0.861
20	8.80	0.808	81.4	7.10	3.16	73.2	0.886
23	11.63	1.064	70.8	7.10	3.17	73.0	0.886
26	14.87	1.368	62.6	7.10	3.17	73.3	0.886
30	19.79	1.816	54.2	7.10	3.17	73.1	0.886
35	26.83	2.464	46.4	7.09	3.15	73.0	0.885
40	35.19	3.232	40.6	7.09	3.16	73.1	0.885
45	44.54	4.088	36.2	7.10	3.16	73.2	0.886
50	54.99	5.048	32.4	7.05	3.16	72.9	0.881
56	84.55	6.328	29.0	7.08	3.17	73.2	0.884
62	84.55	7.760	26.2	7.09	3.17	73.1	0.885
68	101.70	9.336	24.0	7.11	3.17	73.3	0.887

The average values of the characteristic constants are:

Capacity constant $K_q = 3.18$.
Speed constant $K_v = 7.07$.
Type characteristic $K_t = 73.1$.

THE TRUMP MFG. CO., SPRINGFIELD, O.

The Trump Mfg. Co. is one of the best known turbine manufacturers, mainly on the foreign market. At the time when European concerns were not willing or prepared to build radial

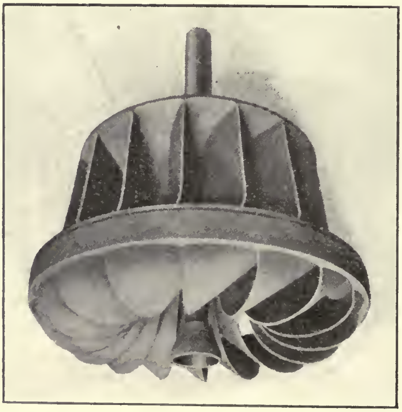

FIG. 17.

TRUMP RUNNER.

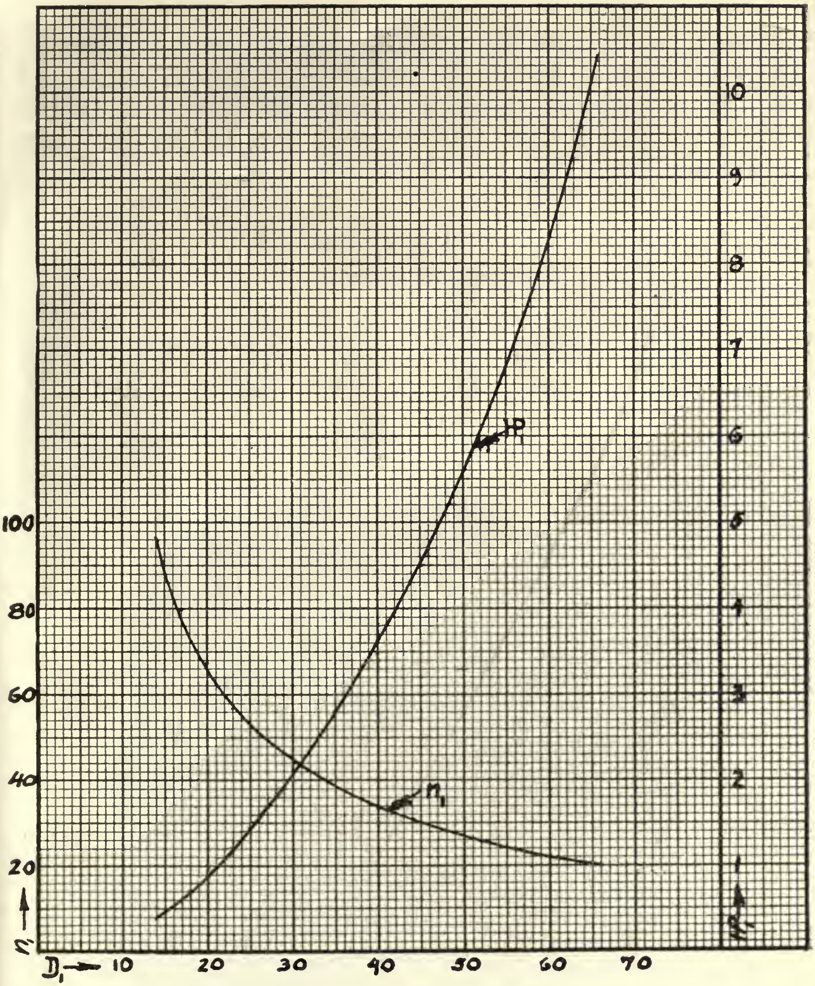

Fig. 18.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY THE. TRUMP MANUFACTURING CO.

TRUMP RUNNER.

inward flow turbines, or were only starting to do so, many of such wheels were installed by the Trump Mfg. Co. all over the European continent. Like the Samson, the Trump runner has steel plate buckets and in form resembles the other American high speed runners.

TABLE NO. VI.

TRUMP RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
14	4.12	0.375	96.2	5.89	3.02	58.9	0.735
17	6.28	0.570	79.2	5.89	3.13	59.8	0.735
20	10.03	0.801	67.4	5.89	3.61	60.4	0.735
23	13.31	1.210	58.6	5.87	3.62	64.5	0.732
26	17.21	1.564	51.0	5.78	3.42	63.7	0.721
30	22.61	2.135	44.4	5.80	3.62	65.0	0.723
35	30.86	2.805	38.2	5.80	3.63	64.1	0.723
40	40.10	3.646	33.6	5.88	3.61	64.2	0.734
44	48.51	4.410	30.6	5.87	3.62	64.3	0.732
48	57.70	5.247	28.0	5.87	3.61	64.2	0.732
52	63.43	6.158	25.8	5.85	3.37	64.1	0.730
56	78.59	7.136	24.0	5.87	3.61	64.1	0.732
61	92.92	8.444	22.0	5.87	3.60	64.0	0.732
66	114.28	10.384	20.4	5.88	3.77	65.7	0.734

The average values of the characteristic runner constants are:

Capacity constant $K_q = 3.52$.
Speed constant $K_v = 5.87$.
Type characteristic $K_t = 63.4$.

RISDON ALCOTT TURBINE CO., MOUNT HOLLY, N. J.

The types of runners manufactured by the Risdon Alcott Turbine Co. are very numerous, due to the fact that this concern is a combine of two turbine manufacturers, the T. H. Risdon Co. and the T. C. Alcott & Son. We shall consider here only the *Alcott High Duty Special*, the *Risdon Double Capacity*, and the *Leviathan* runner.

TABLE NO. VII.

ALCOTT HIGH DUTY SPECIAL RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
10	1.65	0.155	122.2	5.28	2.38	48.2	0.657
12	2.26	0.203	100.8	5.29	2.26	45.4	0.66
13	2.84	0.257	94.2	5.35	2.42	47.7	0.667
15	3.52	0.317	83.4	5.45	2.25	47.0	0.68
18	5.08	0.456	70.2	5.52	2.25	47.4	0.689
21	6.90	0.621	59.8	5.47	2.25	47.2	0.682
24	9.02	0.812	52.0	5.45	2.26	46.8	0.68
27	11.36	1.027	47.0	5.52	2.24	47.7	0.689
30	14.10	1.269	42.0	5.49	2.25	47.3	0.685
36	20.31	1.826	35.2	5.52	2.26	47.6	0.689
42	27.61	2.483	30.0	5.50	2.26	47.3	0.686
48	36.09	3.246	26.2	5.48	2.25	47.2	0.684
54	28.99	2.727	23.4	5.50	1.43	38.7	0.686
60	35.94	3.258	21.0	5.50	1.44	37.9	0.686
66	48.13	4.364	19.4	5.58	1.59	40.5	0.697

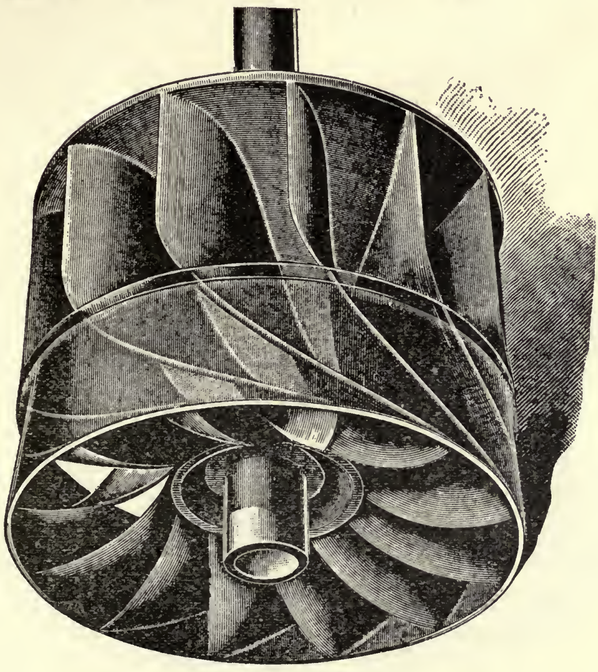

FIG. 19.

ALCOTT HIGH DUTY SPECIAL RUNNER.

TABLE NO. VIII.

RIDSON DOUBLE CAPACITY RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
12	1.17	0.1064	101.0	5.29	1.17	33.	0.66
16	2.34	0.2256	78.4	5.50	1.32	37.2	0.686
20	4.07	0.3856	66.0	5.73	1.50	40.4	0.715
25	6.78	0.5104	54.4	5.95	1.54	38.9	0.742
30	11.00	0.8024	47.2	6.12	1.76	42.3	0.764
36	15.60	1.4032	39.2	6.15	1.73	46.5	0.707
40	18.93	1.8240	35.2	6.18	1.71	47.5	0.077
43	24.51	2.3616	33.0	6.17	1.91	50.7	0.769
50	31.20	2.8288	26.8	5.88	1.80	45.1	0.734
54	38.93	3.536	25.1	5.89	1.92	47.2	0.735
60	47.34	4.5632	22.4	5.80	1.89	47.8	0.723
66	57.86	5.5808	21.6	6.25	1.91	51.1	0.780
72	72.40	6.5664	28.8	5.87	2.01	48.2	0.732

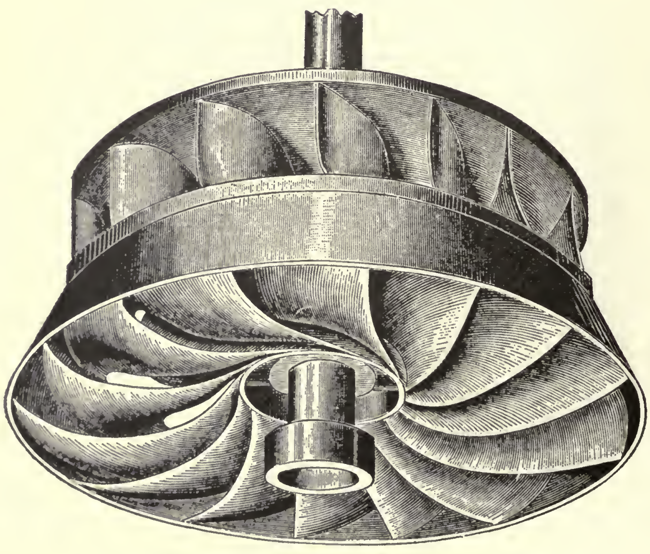

FIG. 20.

RISDON DOUBLE CAPACITY RUNNER.

TABLE NO. IX.

LEVIATHAN RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
18	6.67	0.608	95.0	7.47	2.96	74.	0.932
21	9.08	0.824	81.4	7.45	2.96	74.	0.930
24	11.86	1.080	71.2	7.45	2.95	74.1	0.930
27	15.01	1.368	63.4	7.46	2.96	74.2	0.931
30	18.53	1.688	57.0	7.45	2.96	74.2	0.930
36	26.68	2.424	47.6	7.49	2.96	74.2	0.935
42	36.32	3.304	40.8	7.48	2.95	74.2	0.934
48	47.43	4.312	35.6	7.44	2.97	74.	0.928
54	60.03	5.456	31.6	7.44	2.96	73.9	0.928
60	74.11	6.736	28.6	7.47	2.96	74.3	0.932
66	89.67	8.152	26.0	7.49	2.96	74.3	0.935
72	106.72	9.696	23.8	7.50	2.95	74.1	0.936

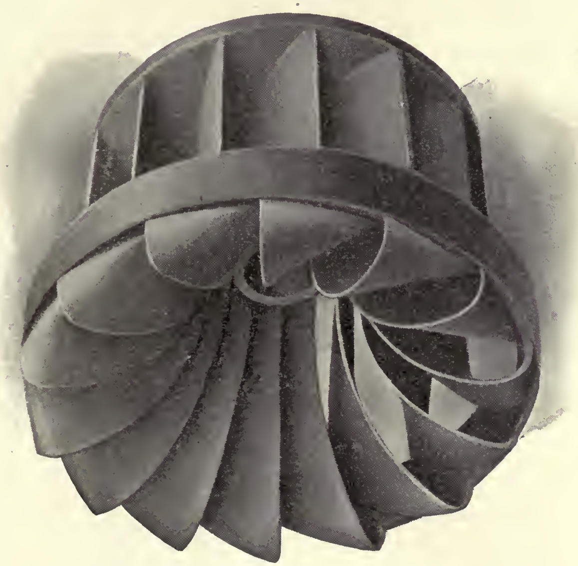

FIG. 21.
LEVIATHAN RUNNER.

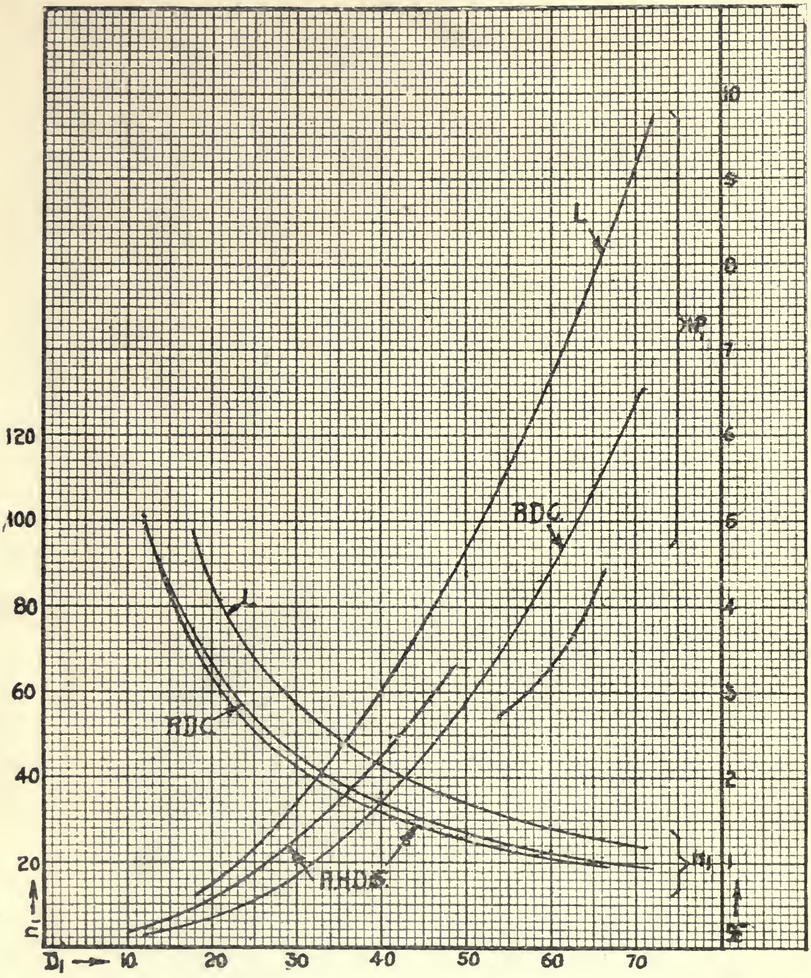

FIG. 22.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY RISDON ALCOTT TURBINE CO.

A. H. D. S.—ALCOTT HIGH DUTY SPECIAL RUNNER.
R. D. C.—RISDON DOUBLE CAPACITY RUNNER.
L.—LEVIATHAN RUNNER.

From Table No. IX it appears that, similar to the Improved New American runners, the nominal diameters are larger than the real mean diameters. The values of the speed constant K_v seem to be too large, and those of the capacity constant too small for the large values of K_t.

The average values of the characteristic runner constants are:

	Alcott High Duty Special	Risdon Double Capacity	Leviathan
Capacity constant K_q..............	2.25	1.7	2.96 (?)
Speed constant K_v................	5.46	5.9	7.47 (?)
Type characteristic K_t............	46.7	43.8	74.1

The mean values of the Alcott High Duty Special have been calculated from values of runner diameters up to 48″. The runners 54″, 60″ and 66″ diameter have a reduced capacity.

MORGAN SMITH CO., YORK, PA.

The Morgan Smith Co. manufactures the noted *McCormick* and *New Success* turbines. Recently a new type has been put on the market by this company under the name of the *Smith* turbine. This new runner, as it can be seen from Table No. XII, has attained somewhat higher values for K_t than those of the Improved New American, which was the leading runner in this respect until the Smith Turbine appeared.

TABLE NO. X.

MC CORMICK RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
9	1.52	0.138	132.8	5.22	2.70	49.2	0.652
12	2.65	0.240	99.6	5.20	2.45	48.7	0.649
15	4.22	0.382	79.6	5.20	2.70	49.2	0.649
18	6.17	0.560	64.4	5.05	2.74	48.2	0.630
21	8.73	0.792	61.2	5.61	2.85	54.5	0.700
24	11.53	1.046	50.6	5.30	2.88	51.8	0.661
27	14.61	1.308	47.2	5.55	2.88	54.0	0.630
30	17.59	1.595	41.6	5.44	2.82	52.6	0.680
33	21.53	1.951	36.2	5.20	2.85	50.6	0.652
36	24.71	2.240	35.4	5.55	2.75	53.0	0.693
39	29.05	2.634	31.0	5.28	2.75	50.2	0.657
42	35.67	3.233	30.0	5.49	2.92	54.0	0.687
45	37.98	3.444	27.4	5.37	2.70	50.8	0.670
48	42.85	3.885	24.6	5.14	2.78	48.5	0.694
51	48.79	4.433	24.8	5.51	2.69	52.2	0.687
54	57.44	5.208	22.8	5.38	2.84	52.1	0.671
57	64.45	5.843	22.2	5.51	2.86	53.7	0.688

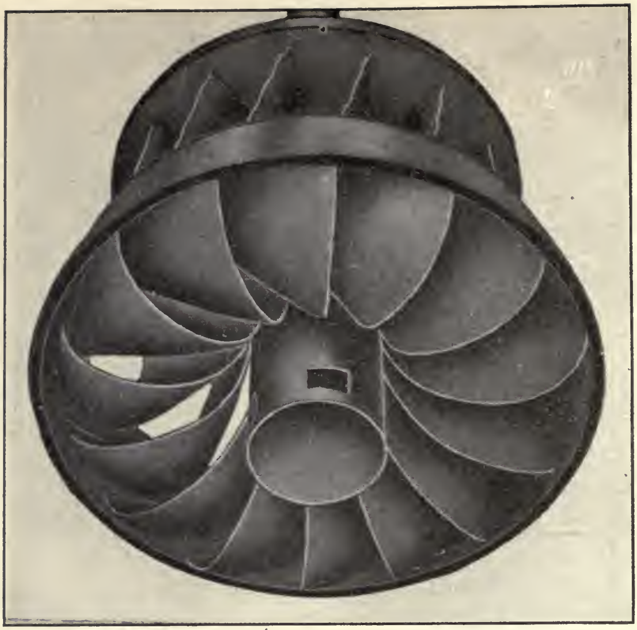

FIG. 23.
MC CORMICK AND NEW SUCCESS RUNNER.

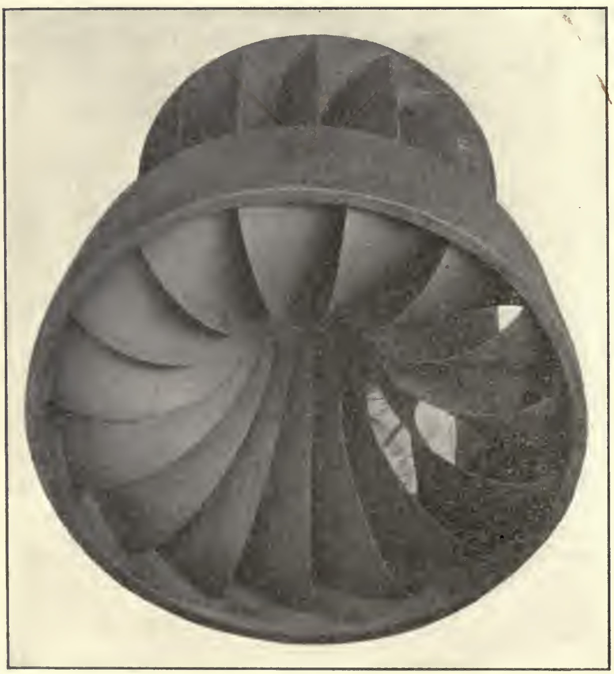

FIG. 24.

TABLE NO. XI.

NEW SUCCESS RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
9	1.47	0.133	146.	5.72	2.62	53.1	0.713
12	2.57	0.233	109.4	5.72	2.57	52.7	0.713
15	4.09	0.370	87.4	5.72	2.62	53.2	0.713
18	5.99	0.543	70.8	5.55	2.66	52.2	0.693
21	8.47	0.768	67.2	6.17	2.76	58.8	0.770
24	11.19	1.014	55.6	5.81	2.80	55.9	0.724
27	14.17	1.284	51.8	6.10	2.79	58.8	0.761
30	17.06	1.547	45.6	5.96	2.73	56.7	0.744
33	22.74	1.898	39.8	5.72	3.00	54.9	0.713
36	23.97	2.173	38.8	6.07	2.64	59.2	0.758
39	28.18	2.554	34.0	5.78	2.87	54.3	0.721
42	34.60	3.136	33.0	6.04	2.83	58.4	0.754
45	36.84	3.339	30.0	5.88	2.61	54.7	0.734
48	41.57	3.768	28.6	5.98	2.64	55.6	0.746
51	47.29	4.290	27.2	6.05	2.61	56.4	0.755
54	55.12	5.051	25.0	5.88	2.75	56.2	0.734
57	62.51	5.667	24.4	6.07	2.77	58.1	0.757
60	81.03	7.340	22.6	5.90	3.24	61.3	0.736
66	98.04	8.888	20.4	5.88	3.23	60.8	0.734
72	123.37	11.187	18.6	5.83	3.42	62.2	0.728
84	167.99	14.430	16.0	5.86	3.42	60.8	0.931

TABLE NO XII.

SMITH RUNNER.

D_1	Q_1	$H\text{-}P_1$	N_1	K_v	K_q	K_t	K'_v
12	3.68	0.338	138.6	7.28	3.73	80.5	0.910
15	5.81	0.530	110.8	7.26	3.73	80.7	0.907
18	8.28	0.760	92.4	7.23	3.67	80.5	0.903
21	11.29	1.034	79.2	7.25	3.69	80.5	0.905
24	14.73	1.353	69.2	7.25	3.67	80.5	0.905
27	18.63	1.710	61.6	7.25	3.68	80.7	0.905
30	23.01	2.113	55.4	7.25	3.68	80.5	0.905
33	27.85	2.558	50.6	7.30	3.68	80.9	0.912
36	33.19	3.047	40.4	7.29	3.68	81.0	0.911
39	38.91	3.573	42.6	7.25	3.69	80.5	0.905
42	45.10	4.143	39.6	7.26	3.69	80.5	0.906
45	51.76	4.753	37.0	7.25	3.67	80.8	0.905
48	58.43	5.362	34.6	7.25	3.65	80.2	0.905
51	66.56	6.111	32.6	7.26	3.68	80.7	0.906
54	74.90	6.850	30.8	7.28	3.69	80.7	0.910
57	83.12	7.632	29.2	7.26	3.70	80.7	0.906
60	92.10	8.456	27.8	7.30	3.68	80.7	0.912
63	101.56	9.324	26.4	7.25	3.67	80.7	0.905
66	111.45	10.233	25.2	7.28	3.67	80.7	0.910
72	132.64	12.178	23.0	7.22	3.68	80.3	0.902

The average values of the characteristic runners are:

	McCormick	New Success	Smith
Capacity constant K_q	2.96	2.8	3.68
Speed constant K_v	5.35	5.88	7.26
Type characteristic K_t	51.4	55.	80.6

THE WELLMAN-SEAVER-MORGAN COMPANY, CLEVELAND, OHIO.

The "Standard" runner manufactured by the Wellman-Seaver-Morgan Co. is the *Jolly McCormick* runner. A table of the characteristics and curves of this runner has been omitted, as they are exactly the same as those of the McCormick runner, manufactured by the S. Morgan Smith Co. See Table No. X, and curves, Fig. 25.

The Wellman-Seaver-Morgan Co. manuafctures also a "Special" runner with increased capacity and increased speed. Judging from the test of a 33″ turbine, the values of the characteristic runner constants are:

Capacity constant $K_q = 3.2$.
Speed constant $K_v = 6.49$.
Type characteristics $K_t = 68.2$.

Lately this company produced another remarkable runner with the following characteristic constants:

Capacity constant $K_q = 3.6$ (3.96).
Speed constant $K_v = 6.47$.
Type characteristics $K_t = 78.5$.

whereby the value $K_q = 3.6$ corresponds to 80% efficiency, which value was assumed as basis for all other runners. The value $K_q = 3.96$, corresponds to the actual discharge $Q_1 = 21$ and actual best efficiency 86%.

Arranging now the various runner types according to their type characteristics, we will have answered the question, how far the different concerns have come in reference to the aim of *highest capacity and highest speed with good efficiency.* Table No. XIII gives both the mean and the maximum values of K_t which were reached by the various runners and the corresponding capacity and speed constants K_q and K_v.

As European engineers and European text books frequently refer to the American high speed runners, and as it appears that the information they have regarding the same is very inaccurate,

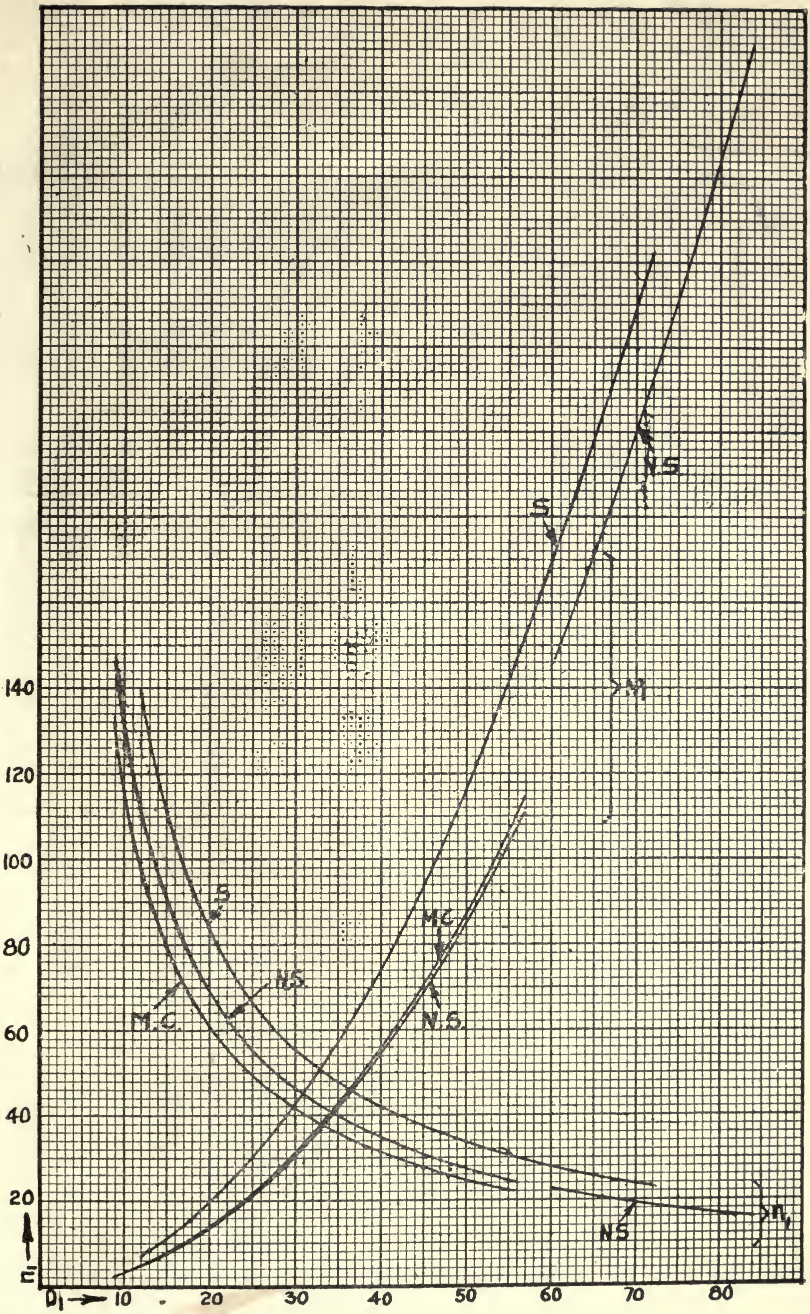

FIG 25.

SPECIFIC SPEED AND SPECIFIC POWER OF THE RUNNERS MANUFACTURED BY S. MORGAN SMITH CO.

M. C.—MC CORMICK RUNNER.
N. S.—NEW SUCCESS RUNNER.
S.—SMITH RUNNER.

TABLE XIII.

NAME OF RUNNER TYPE	MANUFACT'ED BY	FOOT SYSTEM: Capacity Const. $K_q = \frac{Q_1}{D_1^2}$		Speed Const. $K_v = \frac{\pi D_1 N}{60 \sqrt{H}}$		Type Character'c $K_t = N\frac{\sqrt{H\text{-}P}}{H \sqrt[4]{H}}$		METRIC SYSTEM: Capacity Const. $K_q = \frac{Q_1}{D_1^2}$		Speed Const. $K_v = \frac{\pi D_1 N}{60 \sqrt{H}}$		Type Character'c $K_t = N_s = \frac{N \sqrt{H\text{-}P}}{H \sqrt[4]{H}}$		$K'_v = \frac{v_1}{\sqrt{2gH}} = \frac{K_v}{\sqrt{2g}}$	
		MEAN	MAX	MEAN	MAX	MEAN	MAX	MEAN	MAX	MEAN	MAX	MEAN	MAX	MEAN	MAX
Smith	S. Morgan Smith Company	3.68	3.68	7.26	7.29	80.6	81	2.032	2.032	4.008	4.025	358.5	360	0.905	0.908
Improved New American	The Dayton Globe Iron Works Co.	3.43	3.39	7.1	7.3	79	81	1.893	1.872	3.92	4.03	351.2	360	0.885	0.91
Leviathan	Risdon Alcott Turbine Company	2.96	2.96	7.47	7.49	74.1	74.3	1.634	1.634	4.125	4.13	329.5	330.5	0.931	0.934
Improved Samson	The James Leffel Mfg. Company	3.18	3.17	7.07	7.11	73.1	73.3	1.755	1.75	3.902	3.93	325	326	0.881	0.886
Jolly McCormick Special	The Wellman-Seaver-Morgan Co.	3.2		6.49		68.2		1.767		3.58		303.5		0.809	
Victor Increased Capacity	The Platt Iron Works Co.	3.59	3.60	6.1	6.18	66.6	67.2	1.982	1.987	3.368	3.41	296	299	0.761	0.77
Victor Standard Capacity	The Platt Iron Works Co.	3.26	3.27	6.1	6.18	63.5	64.2	1.80	1.805	3.368	3.412	282.5	285.5	0.761	0.77
Trump	The Trump Mfg. Company	3.52	3.77	5.87	5.88	63.4	65.7	1.943	2.081	3.24	3.245	282	292.2	0.729	0.733
New Success	S. Morgan Smith Company	2.75	3.42	5.88	5.83	55	62.2	1.518	1.888	3.245	3.22	244.5	276.8	0.733	0.727
New American	The Dayton Globe Iron Works Co.	2.8	2.85	5.6	5.75	54.1	56.8	15.45	1.573	3.09	3.175	240.5	252.6	0.69	0.717
McCormick and Jolly McCormick	S. Morgan Smith Co. and The Wellman-Seaver-Morgan Co.	2.8	2.85	5.35	5.61	51.4	54.5	1.545	1.573	2.954	3.102	228.6	242.5	0.667	0.699
Alcott High Duty Special	Risdon Alcott Turbine Company	2.25	2.38	5.46	5.28	46.7	48.2	1.242	1.314	3.015	2.915	207.8	214.3	0.681	0.658
Risdon Double Capacity	Risdon Alcott Turbine Company	1.7	1.91	5.9	6.25	43.8	51.1	0.938	1.055	3.256	3.45	195	227.5	0.735	0.779

the runner characteristics have been given also in the metric system.

For 1 (ft.) = 0.30479 (m), 1 (cub. ft.) = 0.028317 (cub. m.), 1 (H-P) = 1.01385 (cheval vapeur) = 1.01385 (metric H-P).

The following conversion constants are to be used, when converting from the foot system into the metric system.

	(in metric system)	(in foot system)
Q_1	$= \dfrac{Q^{(m^3)}}{\sqrt{H^{(m)}}}$	$= 0.0513\, Q_1 = \dfrac{Q^{(ft^3)}}{\sqrt{H^{(ft)}}}$
N_1	" $= \dfrac{N^{(rpm)}}{\sqrt{H^{(m)}}}$	" $= \dfrac{1}{0.552} N_1 = \dfrac{N^{(rpm)}}{\sqrt{H^{(ft)}}}$
$H\text{-}P_1$	" $= \dfrac{H\text{-}P^{(metric)}}{H^{(m)}\sqrt{H^{(m)}}}$	" $= 6.0246\, H\text{-}P_1 = \dfrac{H\text{-}P}{H^{(ft)}\sqrt{H^{(ft)}}}$
K_q	" $= \dfrac{Q_1^{(m^3)}}{D_1^{2(m)}}$	" $= 0.552\, K_q = \dfrac{Q_1^{(ft^3)}}{D_1^{2(ft)}}$
K_v	" $= \dfrac{\pi D_1^{(m)} N^{(rpm)}}{60\sqrt{H^{(m)}}}$	" $= 0.552\, K_v = \dfrac{\pi D_1^{(ft)} N^{(rpm)}}{60\sqrt{H^{(ft)}}}$
K_t	" $= N_s = \dfrac{N^{(rpm)}\sqrt{H\text{-}P^{(metric)}}}{H^{(m)}\sqrt[4]{H^{(m)}}}$	" $= 4.447\, K_t = \dfrac{N^{(rpm)}\sqrt{H\text{-}P}}{H^{(ft)}\sqrt[4]{H^{(ft)}}}$

EXAMPLE:—36″ Smith runner; see Table XII.

	Foot system.	Metric system.
Q_1	$= 33.19$	$0.0513 \times 33.19 = 1.703$
N_1	$= 46.4$	$1/0.552 \times 46.4 = 84.1$
$H\text{-}P_1$	$= 3.047$	$6.0246 \times 3.047 = 18.1$
K_q	$= 3.68$	$0.552 \times 3.68 = 2.032$
K_v	$= 7.29$	$0.552 \times 7.29 = 4.025$
K_t	$= 81.0$	$4.447 \times 81. = 360.$

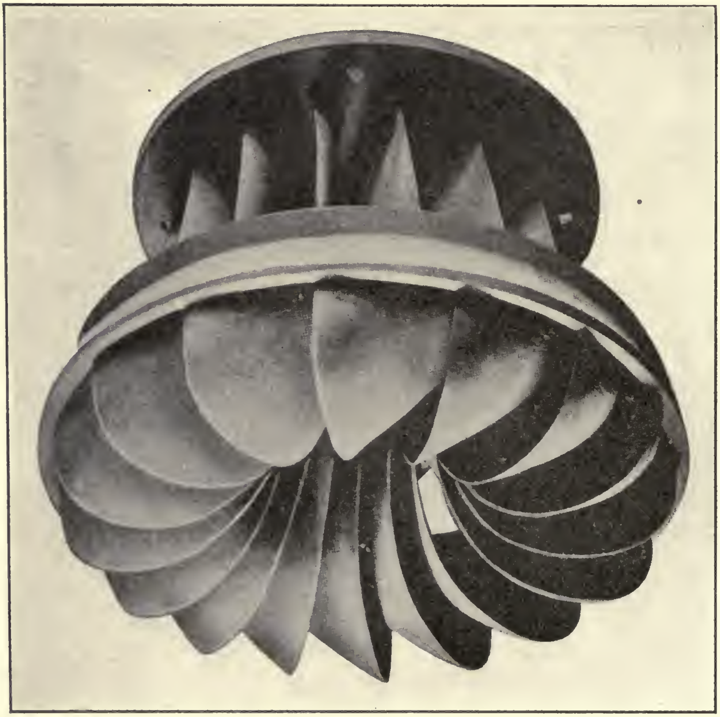

FIG. 26.

JOLLY MC CORMICK RUNNER.

For clearness two sets of curves have been drawn, showing the specific power and speed of the various runner types. In Fig. 27 the curves of the Improved Samson and Trump runner have been omitted, as they would interfere with those of other runners.

The curve of the Improved Samson runner, as can be seen from the values of K_q in Table V, would almost coincide with that of the Victor Standard Capacity runner. The curve of the Trump runner with that of the Victor Increased Capacity. The curve of the Leviathan runner has been drawn as dotted line, because the nominal diameters of this runner type seem to be larger than the real mean diameters and a correction, like with the Improved New American, could not be made for lack of information. Judging from the value of K_t the curve should be in neighborhood of those for the Smith and Improved New American runners.

For the same reasons, the speed curves of the Leviathan, Improved Samson and Trump runners have been omitted in Fig. No. 28.

ABBREVIATIONS.

S = Smith.
I. N. A. = Improved New American.
L. = Leviathan.
V. I. C. = Victor Increased Capacity.
V. S. C. = Victor Standard Capacity.
N. S. = New Success.
N. A. = New American.
M. C. = McCormick.
A. H. D. S. = Alcott High Duty Special.
R. D. C. = Risdon Double Capacity.

At the end it may be emphasized that it was not the intention of the writer to decide which runner type is *best*. To endeavor to answer such a question would be absolutely wrong. There can not be a runner which would be best for all conditions. In many cases the best efficiency will be the deciding factor, but very frequently the variation of the efficiency with the variation of load, and sometimes the maximum capacity or the maximum speed will

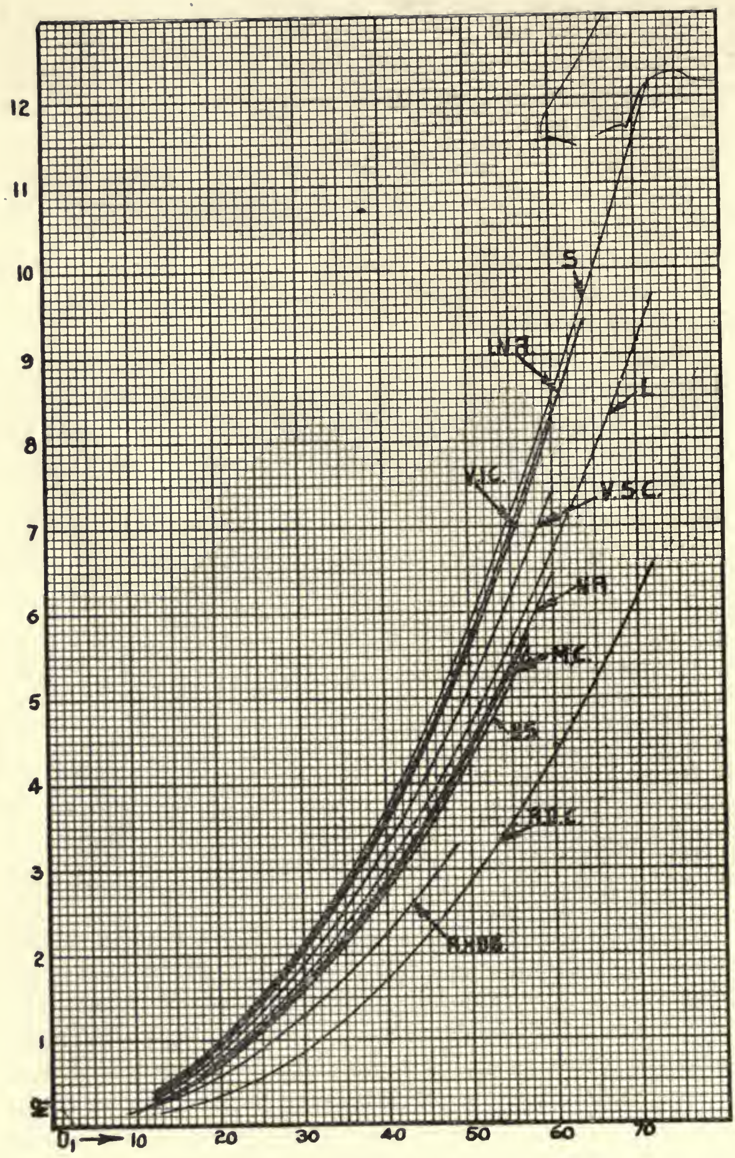

Fig. 27.

SPECIFIC POWER OF THE AMERICAN STANDARD HIGH SPEED RUNNERS.

determine which is the best runner for a given case. Not seldom, for merely technical reasons, the best runner may be one which for capacity, speed and efficiency occupies a minor position among the other runners.

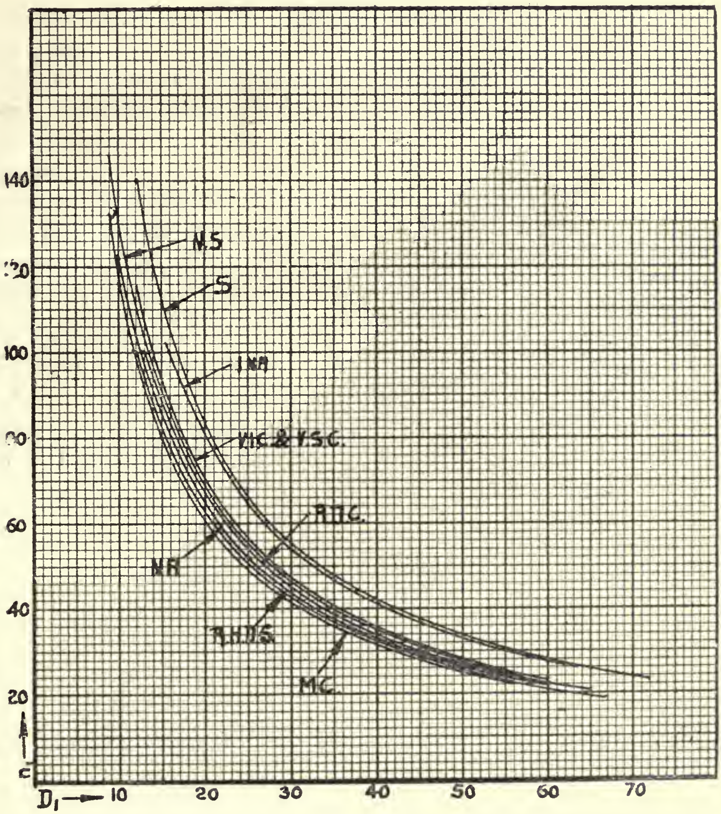

FIG. 28.

SPECIFIC SPEED OF THE AMERICAN STANDARD HIGH SPEED RUNNERS.

ALLIS-CHALMERS CO., MILWAUKEE, WIS.

The manufacture of hydraulic turbines was begun by the Allis-Chalmers Company only six years ago. Following in the beginning the European principle, turbines were designed to suit given conditions and requirements in every instance. But the advantage of standard turbines being fully appreciated, long and exhaustive studies have been made in this direcion by the company's engineers.

At the present time, the developing work on Allis-Chalmers standard turbine types is practically completed. In order that all ordinary combinations of speed and capacity may be covered by these "standards," the company is building six different types of radial inward-flow runners for $K_t = 13$ to about 80, and two types of impulse-wheel buckets. Here only the high speed runner, Type F, interests us. This type was designed to have at least a type characeristic $K_t = 68$. The first runner of this type, a 30-in. runner, was tested in the Holyoke testing flume after the first publication of this article in the summer of 1909. The result of this test, although within the expectations of the engineers, was far beyond the anticipations of the company. The following data obtained from the test are interesting.

At best efficiency, 82.5%, the turbine developed power at the rate $H\text{-}P_1 = 2.28$ with a speed $N_1 = 52$. Thus the values of the runner constants are:

Allis-Chalmers "Type F."

$$K_q = 3.89$$
$$K_v = 6.8$$
$$K_t = 78.7$$

With increased speed, the efficiency went down very slowly, but the output was increased. At speed proportional to $N_1 = 59$, the power was proportional to $H\text{-}P_1 = 2.34$. Thus

$$K_q = 3.80$$
$$K_v = 7.72$$
$$K_t = 90.4$$

SECOND SECTION

A RATIONAL METHOD OF DETERMINING THE PRINCIPLE DIMENSIONS OF WATER-TURBINE RUNNERS.

By S. J. Zowski, Assistant Professor of Mechanical Engineering.

The principal dimensions of a water turbine runner are determined from the required speed and capacity and the available head of water.

Let $D =$ the mean runner diameter, in feet.
$v =$ the corresponding peripheral speed, in feet per second.
$N =$ the required rotative speed, in revolutions per minute.
then $v = \pi DN/60$ and $D = 60v/\pi N$ ....(1)

so that when the proper peripheral speed to give good hydraulic performance is known the diameter of the runner follows therefrom.

Let us assume that the runner is designed in such a way that at its best speed the water discharges from the runner buckets in planes going through the axis of rotation; this is a condition which the turbine designer should always attempt to secure, in order to avoid helical stream lines in the draft-tube. Then the best peripheral speed is given by a simple formula. Denoting the bucket angle by β, the guide-vane angle by α, as in Fig. 1, and the hydraulic efficiency by e_n, the formula is:

$$v = \sqrt{e_h g H} \times \sqrt{\frac{\sin(\beta - \alpha)}{\sin \beta \cos \alpha}} = K_v \times \sqrt{H} \qquad (2)$$

where

$$K_v = \sqrt{e_h g} \times \sqrt{\frac{\sin(\beta - \alpha)}{\sin \beta \cos \alpha}} = \text{speed constant} \qquad (3)$$

The curves in Fig. 2 give the values of the second radical for several constant values of bucket angle β with varying values of

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guide vane angle α. The following limits for α and β appear reasonable.

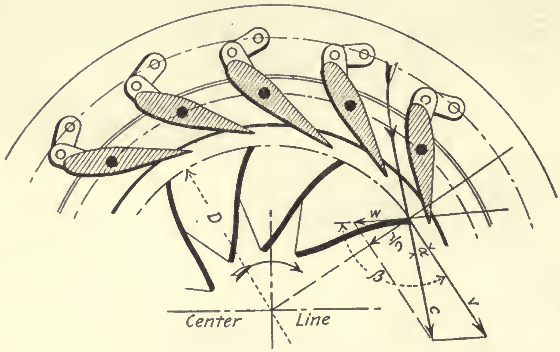


FIG. I.—SECTION THROUGH RADIAL INWARD-FLOW TURBINE SHOWING RELATION BETWEEN BUCKET AND GUIDE-VANE ANGLES.

For a pronounced low-speed turbine $\beta = 60°$. $\alpha = 20°$; then

$$\sqrt{\frac{\sin(\beta - \alpha)}{\sin\beta\cos\alpha}} = 0.88$$

For a pronounced high-speed turbine $\beta = 135°$, $\alpha = 40°$; then

$$\sqrt{\frac{\sin(\beta - \alpha)}{\sin\beta\cos\alpha}} = 1.367$$

For simplicity we will assume that medium-speed runners ($\beta = 90°$) show a hydraulic efficiency of 84% (giving $\sqrt{e_h g} = 5.198$), and other types of runner an efficiency of 83% (giving $\sqrt{e_h g} = 5.167$). These values are by no means taken too high for runners of fair design and construction. Then the speed constant has for the different types the following values:

Type of Runner	Speed Constant K_v	
Low-speed ($\beta = 60°$ to $90°$)	4.588 to 5.198	(4)
Medium-speed ($\beta = 90°$)	5.198	
High-speed ($\beta = 90°$ to $135°$)	5.198 to 7.006	

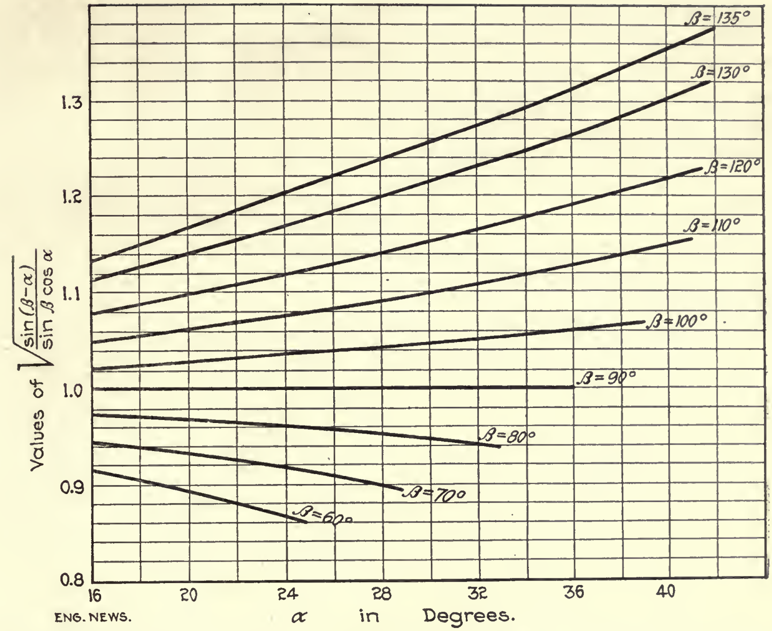


FIG. 2.—CURVES FOR FINDING THE NORMAL PERIPHERAL SPEED OF A TURBINE RUNNER FOR GIVEN BUCKET AND GUIDE-VANE ANGLES.

The ordinates give values of

$$\sqrt{\frac{\sin(\beta - a)}{\sin \beta \cos a}}$$

The speed-constant is

$$K_v = \sqrt{e_h g} \times \sqrt{\frac{\sin(\beta - a)}{\sin \beta \cos a}}$$

where e_h = hydraulic efficiency; g = gravity constant.

The desired peripheral speed of the wheel in feet per second is

$$v = K_v \sqrt{H}$$

where H = effective hydraulic head in feet.

For very high heads, which naturally will require low-speed runners, it will be wise not to approach the minimum value of β, but to remain in the neighborhood of 90°, for the following reason: The smaller the angle β, or to be more exact, the smaller the ratio β/α, the smaller is the pressure-head under which the water passes from the guide case into the runner buckets. This reduced pressure will facilitate the separation of the air that is contained in the water, and thus it will facilitate honey-combing of the runner and guide case, so often observed even in turbines otherwise most carefully designed and highly finished.

Substituting the values of K_v in eq. (1) we obtain the following simple formulas for the runner diameters:

Type of Runner	Formula for Diameter	
Low-Speed	$\frac{87 \text{ to } 99}{N} \times \sqrt{H}$	
Medium-Speed	$\frac{99}{N} \times \sqrt{H}$	(5)
High-Speed	$\frac{99 \text{ to } 134}{N} \times \sqrt{H}$	

As far as speed alone is concerned, any diameter within the above wide limits could be used. The required capacity, however, will limit the choice considerably. The following considerations deal with the influence of capacity.

Let n = number of buckets.
n' = number of guide vanes.
t = thickness of bucket edge.
t' = width of the eddy caused by the guide vane tips and measured on the runner circumference (see Fig. 3).

Then the actual entrance area is:

$$B\left(\pi D - \frac{n\,t}{\sin\beta} - n'\,t'\right) = \pi\,B\,D\left(1 - \frac{n\,t}{\pi\,D\,\sin\beta} - \frac{n'\,t'}{\pi D}\right) = \pi K_1 K_2 D^2 \quad (6)$$

where

$$K_1 = \left(1 - \frac{n\,t}{\pi\,D\,\sin\beta} - \frac{n'\,t'}{\pi\,D}\right) \quad (7)$$

and

$$K_2 = \frac{B}{D} \quad (8)$$

As to the number of buckets used differences of practice will be found among turbine builders. While a few of them use in

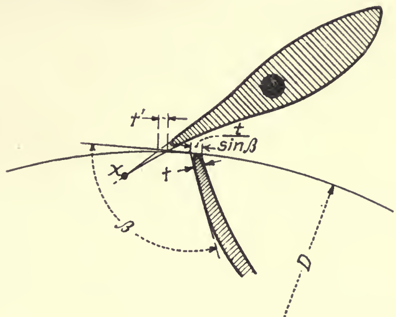


FIG. 3.—SECTION THROUGH GUIDE-VANE AND BUCKET TIP.

every case as large a number of buckets as possible, the majority put into a high-speed runner fewer buckets than into a low-speed runner. The following empirical formulas, in which D is expressed in inches, will give satisfactory results.

Type of Runner	Approx. Number of Buckets	
Low-Speed	$n = 3.7\sqrt{D}$	
Medium-Speed	$n = 3.0\sqrt{D}$	(9)
High-Speed	$n = 2.2\sqrt{D}$	

The number of guide-vanes is very often determined by the simple rule that in every case a few more guide-vanes than buckets should be put in (i. e., $n' = 1.1\, n$ to $1.3\, n$). This rule, however, gives low-speed runners too many guide-vanes, thereby (on account of the small angles a which are used in low-speed turbines) the gate openings become too small and correspondingly the frictional surfaces become relatively large. Therefore it is proper to take account of the guide-vane angle in choosing the number of vanes. The following empirical formulas will give good results. Again taking D in inches,

Guide-vane angle a	Approx. Number of Guide-vanes	
20° and less	$n' = 2.5\sqrt{D}$	
20° to 30°	$n' = 3.0\sqrt{D}$	(10)
30° to 40°	$n' = 3.5\sqrt{D}$	

Since for manufacturing reasons it is advisable that the number of guide-vanes be even, and possibly divisible by four, it will be best to use an even number of buckets, in order to avoid having more than one bucket edge coincide with a guide-vane tip at the same time. On this basis the curves in Fig. 4 have been drawn. These may be used instead of eq. (9) and (10).

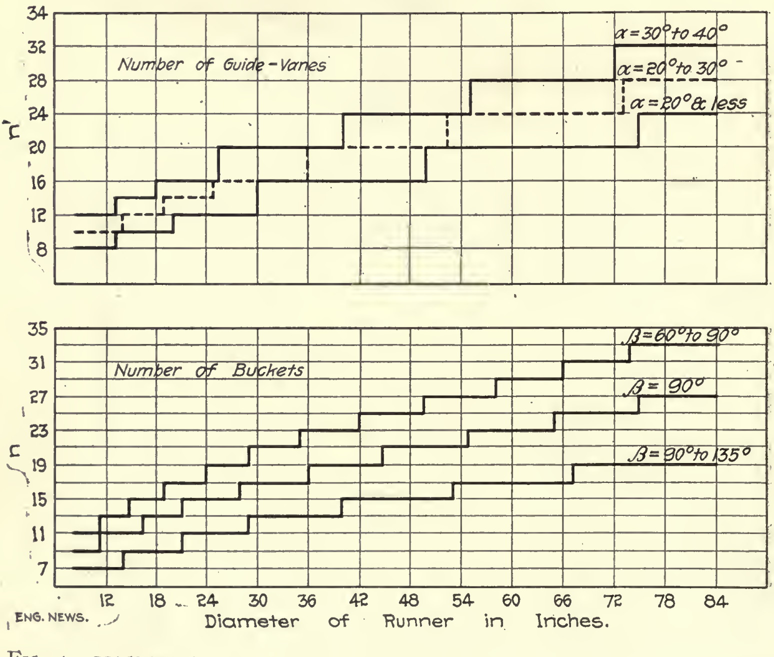


FIG. 4.—DIAGRAM GIVING NUMBER OF BUCKETS AND NUMBER OF GUIDE-VANES FOR DIFFERENT TYPES AND SIZES OF RUNNERS.

The eddies caused by the guide-vane tips should be reduced to a minimum. The writer advises strongly to design the guide case in such a way as to get point *x* (see Fig. 3) outside of the runner circumference. This obviously must be obtained by shaping the vane tips properly and by leaving a sufficient clearance between vane and bucket tips. If this is done the entering streams of water will join in a solid ring, and the effect of the eddies on the capacity of the runner will be nullified. The constant K_1 will then have the value.

$$K_1 = \left(1 - \frac{n\,t}{\pi\, D \sin \beta} \right) \qquad (11)$$

The thickness t varies between ⅛-in. and ¼-in. for steel plate buckets, and between ¼-in. and ⅜-in. for cast buckets.

The following gives the values of K_1 for three different runner sizes, computed from eq. (11):

Runner Diam.	Bucket angle	No. of buckets	Constant K_1: Steel plate buckets		Cast buckets	
D	β	n				
	60°	13		——		0.9154
1 ft.	90°	11	t = ⅛ in.	0.9635	t = ¼ in.	0.9270
	135°	7		0.9671		0.9542
	60°	25		——		0.9600
4 ft.	90°	21	t = ¼ in.	0.9652	t = ¼ in.	0.9652
	135°	15		0.9649		0.9298
	60°	33		——		0.9557
7 ft.	90°	27	t = ¼ in.	0.9744	t = ⅜ in.	0.9616
	135°	19		0.9745		0.9618

For simplicity we shall assume that K_1 has the uniform value 0.93 for all runner types and sizes, with the distinct understanding however that in the final computation the exact value is to be introduced, and also, if necessary, the item $n' \, t'$ be considered.

The capacity of the turbine depends very directly on the ratio B/D, or K_2. As a matter of fact it is this ratio which finally determines the limits for the application of radial inward-flow turbines. Turbine manufacturers are still struggling with the problem of extending these limits in both directions; therefore no definite maximum or minimum values can be given.

Present-day good practice indicates that until further advance is made it is safe to fix the limits of breadth of runner at 1/30 and ½ the diameter. The minimum value depends on the purity of the water. The maximum value which could be allowed depends on the design of the runner, for it is evident that the larger the width of the runner, the more difficult it is to secure the necessary passage area at the point where the water turns from radial to axial direction. In the opinion of the writer it is possible to go somewhat above ½ with the ratio of width to diameter, but then the runner must be bulged out sufficiently.

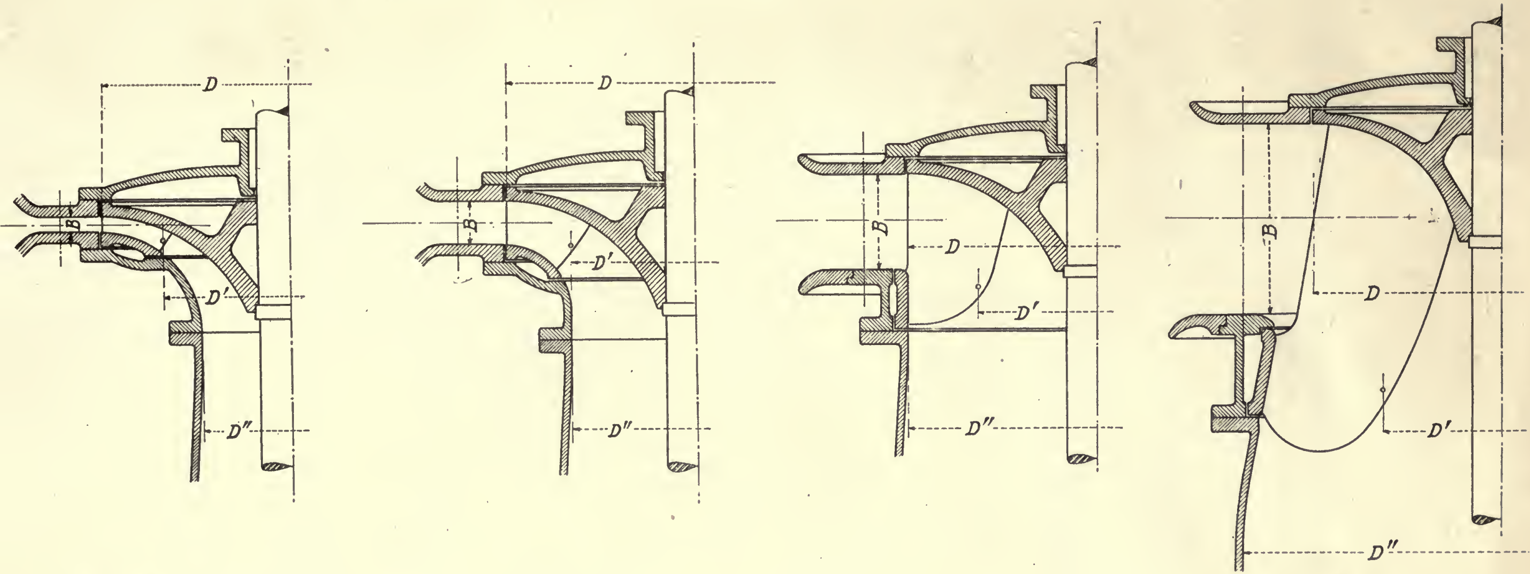
We have classified all runners under three types with reference to speed. It is customary to make a further classification; with reference to capacity. Here also we distinguish three types. The latter classification is based on the proportions of the runner profile, or the ratios of (1) diameter at entrance point of buckets, (2) diameter at exit point of bucket, (3) diameter of neck of draft-tube. These diameters (Fig. 5) will be denoted by D, D', and D''. Their ratios depend mainly on the factor K_2, or B/D.

Type I., or the low-capacity type, comprises all runners in which the draft-tube diameter D'' is less than the bucket exit diameter D', or at most equal to it, and in which B/D lies between 1/30 and ⅛. Type II., the medium-capacity type, comprises runners in which D'' is larger than D' but smaller than the entrance diameter D, and in which B/D lies between ⅛ and ¼. Type III., or the high-capacity type, comprises all runners in which the draft-tube diameter D'' exceeds the bucket entrance diameter D, and in which B/D is between ¼ and ½.

It is self-evident that high heads will require runners of both low-capacity and low-speed type, while low heads will call for high-capacity high-speed runners. In other words, small values of K_2 naturally go with small values of K_v, and large values of K_2 go with large values of K_v. Mistakes in this respect are frequently made by manufacturers of low-head turbines, when they occasionally build a high-head turbine, by giving the runner buckets of such turbines the same entrance angles ($\beta > 90°$) as are used on their low-head turbines. This increases the peripheral speed so much that the runner diameter, and consequently the size of the whole turbine, must be increased considerably in order to obtain the required rotative speed.

A runner is characterized as to its capacity by the so-called capacity constant,

$$K_q = \frac{Q_1}{D^2} = \frac{Q}{\sqrt{H}\,D^2} \qquad (12)$$



Profile A
$\frac{B}{D}=\frac{1}{30}$; $D''<D'<D$.

Profile B
$\frac{B}{D}=\frac{1}{8}$; $D''=D'<D$.

Profile C
$\frac{B}{D}=\frac{1}{4}$; $D''=D>D'$.

Profile D
$\frac{B}{D}=\frac{1}{2}$; $D''>D$.

FIG. 5.—LIMITING PROFILES OF THE THREE TYPES OF RADIAL INWARD-FLOW TURBINE

LOW-SPEED LOW-CAPACITY TYPE.
For High Heads.
$B/D = 1/30$ to $1/8$.
Bucket angle $\beta = 60°$ to $90°$
Vane angle $\alpha = 20°$ and less

MEDIUM-SPEED MEDIUM-CAPACITY TYPE.
For Medium Heads.
$B/D = 1/8$ to $1/4$.
Bucket angle $\beta = 90°$
Vane angle $\alpha = 25°$ to $32°$

HIGH-SPEED HIGH-CAPACITY TYPE.
For Low Heads.
$B/D = 1/4$ to $1/2$.
Bucket angle $\beta = 90°$ to $135°$
Vane angle $\alpha = 30°$ to $40°$

The values of K_q for the different runner types can be found as follows: Area × Speed = Discharge; therefore,

$$Q = \pi K_1 K_2 D^2 c_r \qquad (13)$$

where c_r is the radial component of the entrance velocity c (see Fig. 1). This component is given by

$$c_r = c \sin a = \sqrt{e_h \, g_H} \sqrt{\frac{\sin \beta}{\sin (\beta - a) \cos a}} \sin a = K_3 \sqrt{H} \qquad (14)$$

Combining the last two equations, we get

$$Q = \pi K_1 K_2 K_3 \sqrt{H} D^2 = K_q \sqrt{H} D^2 \qquad (15)$$

or,

$$K_q = \pi K_1 K_2 K_3 \qquad (16)$$

in which

$$K_3 = \sqrt{e_h g} \times \sqrt{\frac{\sin \beta}{\sin (\beta - a) \cos a}} \sin a \qquad (17)$$

In Fig. 6 is drawn a series of curves which give the values of the second radical of eq. (17) for the same angles β and a for which the curves in Fig. 2 were drawn. Multiplying the appropriate ordinate taken from Fig. 6 by $\sqrt{e_h g}$, we obtain K_3. The other coefficients (K_1 and K_2) having been found previously, eq. (16) at once gives the value of the Capacity Constant K_q.

Using the same limiting values as before, to define the several types of runner, we find that the Capacity Constant has the following range:

Type of runner	Range of K_q	
Low-speed low-capacity	0.21 to 0.89	(18)
Medium-speed medium-capacity	0.89 to 2.19	
High-speed high-capacity	2.19 to 4.66	

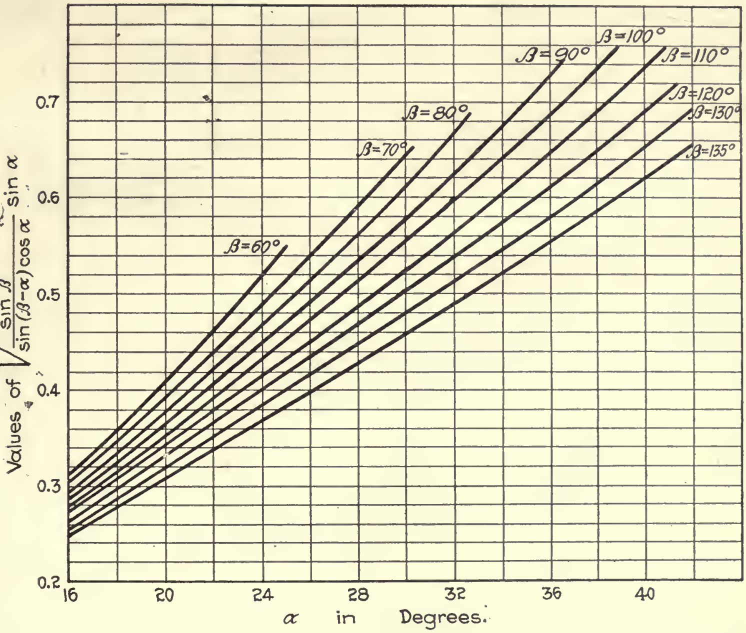


FIG. 6.—CURVES FOR FINDING RADIAL ENTRANCE VELOCITY FOR GIVEN BUCKET AND GUIDE-VANE ANGLES.

The ordinates give values of

$$\sqrt{\frac{\sin \beta}{\sin (\beta - a) \cos a}} \cdot \sin a$$

$$\text{The factor } K_3 = \sqrt{e_h g} \cdot \sqrt{\frac{\sin \beta}{\sin (\beta - a) \cos a}} \cdot \sin a$$

where e_h = hydraulic efficiency; g = gravity constant.

The radial entrance velocity in feet per second is

$$c_r = K_3 \cdot \sqrt{H}$$

where H = effective hydraulic head in feet.

By introducing these values in eq. (12) and solving for diameter we obtain the following simple formulas:

Type of runner	Diam. in terms of discharge per 1-ft. head	
Low-speed low-capacity	(2.20 to 1.06) $\sqrt{Q_1}$	(19)
Medium-speed medium-capacity	(1.06 to 0.67) $\sqrt{Q_1}$	
High-speed high-capacity	(0.67 to 0.46) $\sqrt{Q_1}$	

These formulas, together with eq. (5), will determine which range of diameters can satisfy both the requirements as to speed and capacity. Evidently only those diameters are suitable which satisfy both eq. (5) and eq. (19).

The procedure can be further simplified by the use of a constant which the writer has called Type Characteristic (see Eng. News, Jan. 28, 1909). Its formula is:

$$K_t = \frac{N \times \sqrt{H\text{-}P}}{H \sqrt[4]{H}} = \frac{60\, K_v \times \sqrt{K_q} \times \sqrt{K}}{\pi} \qquad (20)$$

where

$$K = \frac{H\text{-}P}{Q\,H} = \frac{62.42 \times \textit{turbine efficiency}}{550}$$

For a turbine efficiency of 80%, $K = 1/11$, which may be used as a fair average value for the present purpose. With this figure, and the values of K_q and K_v, tabulated previously, we obtain the following ranges for the Type Characteristic:

Type of runner	Type characteristic K_t	
Low-speed low-capacity	12 to 28	(21)
Medium-speed medium-capacity	28 to 44	
High-speed high-capacity	44 to 87	

With these formulas the determination of proper runner type and diameter is very simple. We proceed as follows:

From the given values of horsepower output $H\text{-}P$, speed of revolution N, and hydraulic head H, compute the type characteristic K_t by eq. (20). If the resulting figure is between 12 and 28, a radial inward-flow turbine is possible, and the runners will have to be of the low-speed, low-capacity type, with $B/D = 1/30$ to $1/8$,

$\beta = 60°$ to $90°$, and a profile which will fall between profiles *A* and *B* of Fig. 5.

If the value of K_t is between 28 and 44, the runner will have to be of the medium-speed, medium-capacity type, with $B/D =$ ⅛ to ¼, $\beta = 90°$, and a profile which will fall between profiles *B* and *C* of Fig. 5.

If the value of K_t is between 44 and 87, the runner will have to be of the high-speed, high-capacity type, with $B/D =$ ¼ to ½, $\beta = 90°$ to $135°$, and a profile which will fall between profiles *C* and *D* of Fig. 5.

If the value of K_t is smaller than 12, and it does not seem advisable to make *B* smaller than 1/30 of the diameter, a radial inward-flow turbine is not possible, and an impulse wheel will have to be used.

If the value of K_t is larger than 87, a multiplex turbine must be built. That is to say, a case for which K_t is found to be, say, 174, which is 2×87, would require a quadruplex turbine of which each runner is designed for $K_t = 87$.

Knowing the type of runner it is easy to find the other principal dimensions, as now we can not make a mistake in the choice of the rational values for the different constants in our equations. A few words must be added, however, in reference to the draft-tube diameter D''. The flow velocity c'' at the point where D'' is measured, the "upper draft-tube area," is, in properly designed runners, more or less the same as the flow velocity in the discharge area of the runner. This velocity represents a direct loss; but the loss is partly recovered by the conical lower part of the draft-tube. In low-capacity runners there is no difficulty in reducing the discharge loss to a minimum in the runner itself. In high capacity runners, on the other hand, larger discharge losses must be allowed, as otherwise the runner would have to be bulged out too much. Expressing the discharge loss measured at the upper draft-tube area in parts of the total head, the following values will represent good practice.

Type of runner	Discharge loss in terms of total head	
Low-speed low-capacity	(0.04 to 0.06) H	(22)
Medium-speed medium-capacity	(0.05 to 0.08(0.1)) H	
High-speed high-capacity	(0.08 to 0.15(0.2)) H	

NUMERICAL EXAMPLES.—The following specimen calculations will illustrate the application of the method set forth in this article:

I.—Given $H = 100$ ft.; H-P $= 2500$ HP.; $N = 250$ r. p. m.

Assuming 80% efficiency, $Q = \dfrac{11 \times 2500}{100} = 275$ cu. ft. per sec.

From eq. (20),

$$K_t = \frac{250 \times \sqrt{2500}}{100 \sqrt[4]{100}} = 39.55$$

Comparing this with the sets of values given by eq. (21) we see that the runner has to be of the medium-speed, medium-capacity type, with bucket angle $\beta = 90°$. Therefore, from eq. (5),

$$D = \frac{99}{N} \times \sqrt{H} = \frac{99}{250} \times \sqrt{100} = 3.96\,ft.$$

Take $D = 4$ ft. The number of buckets is 21, from the chart Fig. 4. The thickness of bucket edge, using cast buckets, is ¼-in. The number of guide-vanes (from Fig. 4) is 20, but the guide case shall be designed in such a way that $t' = 0$ giving for the free circumferance $\pi\ D - n\ t = \pi \times 48 - 21 \times \frac{1}{4} = 145.55$ ins. $= 12.16$ ft.

The type characteristic, 39.55, is nearer the upper limit for type II than the lower limit; assume, therefore, $B = \frac{1}{4} D = 1$ ft., and $D'' = D = 4$ ft. Then the free entrance area $1 \times 12.16 = 12{:}16$ sq. ft. Consequently,

$$c_r = \frac{Q}{12.16} = \frac{275}{12.16} = 22.62\ \textit{ft. per sec.}$$

For $\beta = 90°$, $c_r = c = \sqrt{e_h g H.}$ tan α. Hence, assuming

$$e_h = 0.84,\ tan\ a = \frac{c_r}{\sqrt{e_h g H.}} = \frac{22.62}{51.98} = 0\ 435$$

whence $\alpha = 23°\ 30'$.

The upper draft-tube area, if the shaft does not extend into the draft-tube, is $\frac{1}{4} \pi \times 4^2 = 12.57$ sq. ft. Consequently the discharge velocity is

$$c'' = \frac{275}{12.57} = 21.86\ \textit{ft. per sec.}$$

and the discharge loss percentage is

$$\frac{(c'')^2}{2\,g\,H} = \frac{22.86^2}{64.32 \times 100} = 0.0743$$

which value is satisfactory, according to eq. (22).

II.—Given $H = 36$ ft.; $H\text{-}P = 4{,}000$ HP.; $N = 200$ r.p.m., so that $Q = 1{,}222$ cu. ft. per sec. It is required that the turbine be capable of sustaining 15% overload.

The type characteristic is

$$K_t = \frac{200 \times \sqrt{4000}}{36 \sqrt[4]{36}} = 143.5$$

Comparing with eq. (21) we find that we must use a quadruplex turbine, with runners designed for

$$K_t = \frac{143.5}{2} = 71.8$$

This requires Type III, and consequently, by eq. (5),

$$D = \frac{99 \text{ to } 134}{N} \times \sqrt{H}.$$

Since the value of K_t is nearer the maximum than the minimum for Type III, take

$$D = \frac{125 \times \sqrt{H}}{N} = \frac{125 \times \sqrt{36}}{200} = 3.75\,ft. = 45\,ins.$$

Then the speed constant is

$$K_v = \frac{\pi\,D\,N}{60\,\sqrt{H}} = \frac{\pi \times 3.75 \times 200}{60\,\sqrt{36}} = 6.545$$

Assuming $e_h = 0.83$, $\sqrt{e_h g}$ is found to be 5.167 then,

$$\frac{K_v}{\sqrt{e_h g}} = 1.266 = \sqrt{\frac{\sin\,(\beta - a)}{\sin\,\beta\,\cos\,a}}$$

From the curves in Fig. 2, we see that we could use $\beta = 135°$ $a = 31°$; or $\beta = 130°$, $a = 35°\,30'$. We choose the former combination, because the turbine must be capable of carrying overload. For these angles the value of $\sqrt{\frac{\sin\,\beta}{\sin\,(\beta - a)\,\cos\,a}} \times \sin a$ is 0.475, from Fig. 6. Consequently the radial entrance velocity $c_r = 0.475 \times \sqrt{e_h g\,H} = 0.475 \times 5.167 \times \sqrt{36} = 14.706$. The number of buckets, from Fig. 4, is 15. The guide case being designed so that $t' = 0$, the free circumference is

$$\pi D - \frac{n t}{\sin \beta} = \pi\ 45\ \frac{15 \times \frac{1}{4}}{\sin 135^\circ} = 136.03\ ins. = 11.333\ ft.$$

Hence, $11.335 \times B \times 14.706 = 1222/4$, whence $B = 1.823$ ft. Taking $B = 22$ ins., the ratio of width to diameter is $22/45 = 1/2.045$, which value is satisfactory.

Take into consideration the runner nearest the generator and assume that the shaft projecting from this runner into the draft-tube is $10\frac{1}{2}$ ins. in diameter; then the upper draft-tube area in square feet is

$$\left(\frac{\pi D''^2}{4} - \frac{\pi \times 10\frac{1}{2}^2}{4}\right)\frac{1}{144}$$

for D'' taken in inches. Assuming an allowable draft-tube loss of 0.14, we have

$$\frac{(c'')^2}{2g} = 0.14\,H, \text{ or } c'' = \sqrt{2g \times 0.14 \times 36} = 18 \text{ ft. per sec.}$$

the draft-tube velocity. Then the diameter of the draft-tube must be found from

$$Q = \left(\frac{\pi D''^2}{4} - \frac{\pi \times 10.5^2}{4}\right)\frac{1}{144} \times 18 = \frac{1222}{4}$$

or

$$\frac{\pi D''^2}{4} = \frac{1222}{4} \times \frac{144}{18} + \frac{\pi \times 10.5^2}{4} = 2530.59 \text{ sq. ins.}$$

whence $D'' = 56.8$ ins. Using 57 ins. for round numbers, the discharge loss is reduced to $0.138\,H$.

The guide case and the regulating device shall be designed so that maximum gate-opening corresponds to the guide-vane angle $\alpha = 40^\circ$. Assuming for the present that eq. (14) for radial entrance velocity is true up to the maximum gate opening we find with the aid of the curves in Fig. 6 that max. $c_r = 0.618\sqrt{e_h g H}$, whereas at normal gate opening we found $c_r = 0.475\sqrt{e_h g H}$. If we further assume that the efficiency remains unchanged, the discharge at maximum gate-opening 40° will evidently be

$$Q_{max} = \frac{0.618}{0.475} \times Q = 1.3\,Q,$$

or, in other words, the turbine is capable of 30% overload theoretically. Practically the overload capacity will be somewhat smaller, as the assumptions made are not correct. Formula (14) is valid only for the "normal" gate opening; i. e., that for which the turbine was originally designed and at which the water discharges from the runner buckets in planes going through turbine axis. Further, the hydraulic efficiency always is smaller when the gate opening is different from the "normal" gate opening.

This is not the place to go any further into the difficult question of the variation of speed, discharge and efficiency with varying gate opening. Suffice it to state that, judging from actual tests we may expect in the case at hand that the required overload, being only half of the overload as figured before, will certainly be obtained, very likely before the gates are opened to the full 40° angle.

It is hoped that the above outlined method of computing water turbine runners will be useful to many engineers and will help to eliminate turbines of irrational types and proportions. It goes without saying that the values of the different constants indicated by the writer, must not be adhered to too closely, and that the boundaries between the different types are not sharp, but may be changed more or less according to the designer's preference. Thus it would not make much difference whether a runner of $K_t = 43$ were designed with $D'' = D$, or with D'' somewhat larger than D.

THE TYPE CHARACTERISTIC OF IMPULSE WHEELS AND ITS USE IN DESIGN.

By S. J. Zowski.

A previous article by the author, entitled "A Comparison of American High-Speed Runners for Water Turbines,"* the first of this series, gave the method of deriving a characteristic called the Type Characteristic, by means of which a convenient classification and comparison of existing runners and runner types is obtained. In a second article, entitled "A Rational Method of Determining the Principal Dimensions of Water-Turbine Runners,"† the second of this series, the convenience of using the Type Characteristic for computing *new* runners was demonstrated. Both articles, however, dealt with the radial inward-flow turbine only. But as the type Characteristic renders equally good service with all other turbine classes, it is proposed in this article to investigate the impulse wheel in a similar way.

The following notation will be used:

d = the actual diameter of the jet.§ In this country only circular nozzles are used, therefore square jets will not be considered at all in this article, although such jets would not change the theory materially.

§ This is not the nozzle diameter, but the actual diameter of the jet at the contraction, if there is one, or in any case at the minimum section.

D = the nominal wheel diameter, *i. e.*, the diameter of the circle tangent to the center line of the jet, which circle might appropriately be called the "impulse circle." Some engineers call this circle the "Pelton circle," in recognition of Mr. Pelton's contributions to the development of the impulse wheel.

c = the velocity of the jet.

v = the peripheral velocity of the wheel, measured on the impulse circle.

H = the net head acting in the nozzle.

$H\text{-}P$ = the power of the wheel, in horsepowers.

N = the rotative speed of the wheel, in revolutions per minute.

Unless specified otherwise, the foot and the second are the units.

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* Engineering News, Jan. 28, 1909, pp. 99-102. Michigan Technic, Jan., 1910.

† Engineering News, Jan. 6, 1910. Michigan Technic, June, 1910.

DERIVATION OF TYPE CHARACTERISTIC.

The jet velocity c may be expressed ni terms of the head as follows:

$$c = f_c \sqrt{2gH} \qquad (1)$$

For nozzles of good design and workmanship, the velocity coefficient f_c may be taken as 0.97. The discharge of the nozzle is

$$Q = \frac{\pi d^2}{4} f_c \sqrt{2gH} = 6.3 f_c d^2 \sqrt{H} \qquad (2)$$

The power developed by Q and H is

$$H\text{-}P = \frac{62.42\, Q H}{550} e = \frac{Q H e}{8.8}$$

where e is the wheel efficiency.

By substitution we obtain:

$$H\text{-}P = 0.716\, e\, f_c\, d^2\, H \sqrt{H}$$

Substituting unity for H gives the power at head of one foot, or the specific power:

$$H\text{-}P_1 = 0.716\, e\, f_c\, d^2 = K_P\, d^2 \qquad (3)$$

where

$$K_P = 0.716\, e\, f_c \qquad (4)$$

In determining the principal turbine dimensions, the wheel efficiency is generally assumed as 80%. Therefore we may substitute 0.8 for e, which with 0.97 for f_c gives $K_P = 0.5565$. Solving eq. (3) with this value, we obtain a convenient formula for figuring the required jet diameter.

$$\left.\begin{aligned} &\text{In feet } d = \sqrt{\frac{1}{K_P}} \sqrt{H\text{-}P_1} = 1.34 \sqrt{H\text{-}P_1} \\ &\text{or in inches, } d = 16.1 \sqrt{H\text{-}P_1} \end{aligned}\right\} \qquad (5)$$

The peripheral velocity of the wheel, measured on the impulse circle, is

$$v = f_v \sqrt{2gH} \qquad (6)$$

For polished or well-finished buckets the coefficient may be taken as $0.485 f_c$, which, with $f_c = 0.97$ equals 0.47.* Since the rotative speed of the wheel, N, is equal to $60\ v/\pi D$, we obtain by substituting the value of v from eq. (6),

$$N = 153.17 f_v \frac{1}{D} \sqrt{H}$$

Putting $H = 1$ in this formula we find the specific speed (speed at one-foot head) to be

$$N_1 = 153.17 f_v \frac{1}{D} = K_N \frac{1}{D} \tag{7}$$

where

$$K_N = 153.17 f_v \tag{8}$$

for $f_v = 0.47$, $K_N = 72$.†

Using this value in eq. (7), we obtain a convenient formula for computing the required wheel diameter:

$$D = \frac{K_N}{N_1} = \frac{72}{N_1} = \frac{72 \sqrt{H}}{N} \tag{9}$$

The expression for the Type Characteristic is

$$K_t = \frac{N \sqrt{H\text{-}P}}{H \sqrt[4]{H}} = N_1 \sqrt{H\text{-}P_1} \tag{10}$$

Substitute for N_1 and $H\text{-}P_1$ their values from eq. (3) and (7). This gives

$$K_t = K_N \sqrt{K_P} \frac{d}{D} = 153.17 f_v \sqrt{0.716\ e\ f_c} \frac{d}{D} \tag{11}$$

With $f_v = 0.47$, $f_c = 0.97$ and $e = 0.80$ (or with $K_N = 72$ and $K_P = 0.5565$), we obtain for the type characteristic the simple formula

$$K_t = 53.67 \frac{d}{D} \tag{12}$$

Thus the type characteristic of impulse wheels is a simple function of the ratio of jet and wheel diameter. Therefore, the determination of the limiting values of K_t for impulse wheels means the determination of the limits for the ratio d/D with which reasonable efficiency may be obtained.

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* The value of $f_v \sqrt{2g} = 3.77$ corresponds to the speed constant $K_v = 4.588$ to 7.006 for radial inward-flow turbines.

† The corresponding value of K_N for radial inward-flow turbines is 87 to 134.

LIMITING RATIOS OF JET DIAMETER TO WHEEL DIAMETER.

It is obvious that there will be a certain maximum value for this ratio, because, going for instance to the extreme, a wheel diameter equal to the jet diameter, or $d/D = 1$, would evidently be impossible to use. Therefore there will be a certain *maximum* value for K_t which cannot be exceeded by an impulse wheel using a single jet.

A *minimum* value for K_t does not exist, theoretically, as the ratio d/D could be decreased to any desired amount. In practice, of course, we would come to a limit also in this direction, on account of the limitation as to the size of a wheel. But actual problems would never bring us near this limit, as a wheel of given capacity would never be required to have so low a rotative speed (hence low K_t) as to require wheels of diameter too large to be built or operated successfully. We therefore do not need to consider the question of the minimum value for K_t at all.

The question of the maximum value for K_t, however, is of great importance for practical application, as this will determine the number of wheels or number of nozzles required for the given power and speed. To answer this question, we need, since K_t is a function of d/D, only to find the least wheel diameter that can be used satisfactorily in connection with a jet of given diameter.

The sketches, Figs. 1, 2 and 3, bring out the chief conditions involved. As bucket B progresses from the position shown in Fig. 1, the jet will be split in two parts, one feeding bucket B, the other proceeding with the jet velocity c in its initial direction. When bucket B reaches the position shown in Fig. 3, the water particle which was at m when bucket B separated it from the main jet will have moved to x. Similarly, the water particle which was at n when the bucket separated it from the main jet (Fig. 2) will have moved to y (Fig. 3). These distances are

$$m\,x = m'\,o' \times \frac{c}{v} = m'\,o'\;\frac{f_c}{f_v} = \frac{m'\,o'}{0.485}$$

$$n\,y = n'\,o' \times \frac{c}{v} = n'\,o'\;\frac{f_c}{f_v} = \frac{n'\,o'}{0.485}$$

The curve xyz, Fig. 3, shows thus the end of the jet which has been cut off from the main jet, in its correct instantaneous position at the time when the entrance edge of bucket B is at o. As the wheel moves farther, the separated jet will move, too; when the bucket B reaches, for instance, the position of bucket A in Fig. 1, the curve xyz (Fig. 3) will be moved to $x_1\, y_1\, z_1$ (Fig. 1). These distances are

$$m\, x_1 = m'\, p' \times \frac{c}{v} = m'\, p'\ \frac{f_c}{f_v} = \frac{m'\, p'}{0.485}$$

$$n\, y_1 = n'\, p' \times \frac{c}{v} = n' p'\ \frac{f_c}{f_v} = \frac{n'\, p'}{0.485}$$

$$o\, z_1 = o'\, p' \times \frac{c}{v} = o'\, p'\ \frac{f_c}{f_v} = \frac{o'\, p'}{0.485}$$

It is apparent that, in order to utilize the entire energy contained in the jet, no particle must slip through, without giving up its energy to the buckets. Therefore, the wheel and buckets must be designed so that the last particle of the jet separated by bucket B from the main jet, namely particle z, will *reach* bucket A and will be *fully deflected* by bucket A before this bucket, or that point of the bucket at which the last particle should discharge, leaves the sphere of the jet.

While it is a simple matter to determine the position of the bucket A at the moment when the last particle reaches it, it is very difficult, if possible at all, to determine the position at which the last particle has just completed its flow through the bucket. A great deal of judgment must be used in this respect, and, therefore, special care is advised in all cases where only a short time is left for the bucket to remain in the sphere of the jet after the last particle has entered.

This time can be increased in two ways: (1) By decreasing the time necessary for the last particle to reach the bucket A. This is secured by decreasing the pitch of the buckets, thus shortening the sections of jet cut off by the buckets. (2) By increasing the total length of time during which each bucket remains in the sphere of the jet, or in other words, by increasing the length of the arc oo_1.

In decreasing the pitch or increasing the number of the buckets, however, we soon come to a limit. From Fig. 4, showing two consecutive buckets in section, it is apparent that the smaller the

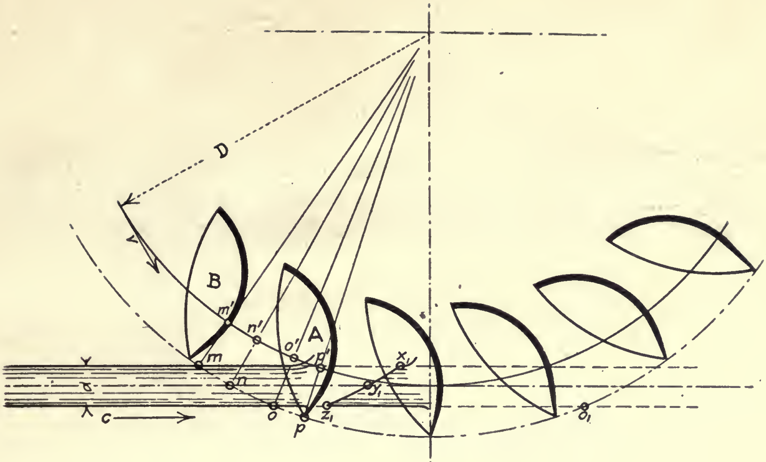

Fig. 1.

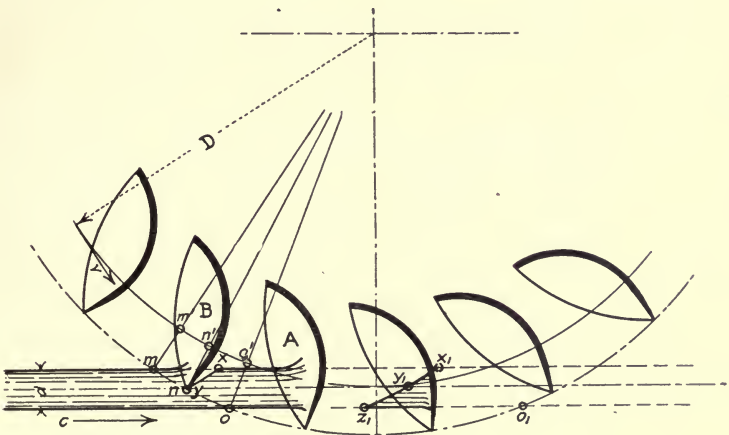

Fig. 2.

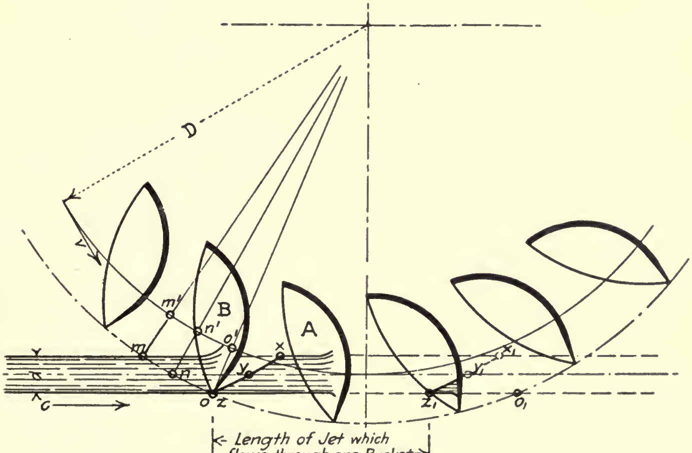

Fig. 3.

FIGS. 1-3. RELATION OF BUCKET AND JET IN THREE DIFFERENT POSITIONS.

pitch, the larger the angle must be in order that the water may freely discharge from the bucket without hitting the following bucket. But with increasing β, the lateral component c' of the

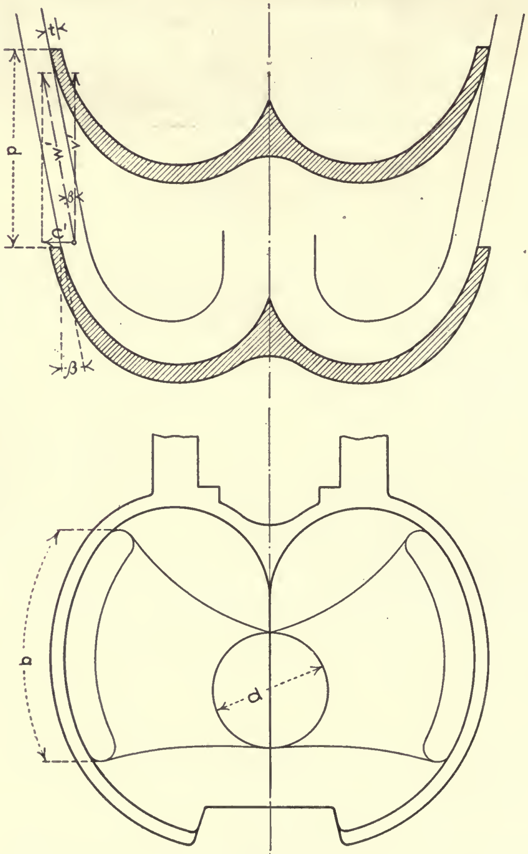

FIG. 4. PATH OF JET DISCHARGING FROM BUCKET.

water velocity grows larger, and this means increased discharge loss $\frac{(c')^2}{2g}$; hence the wheel efficiency is decreased. Efficiency demands that we should make angle β as small and the pitch as large as possible. Allowing a certain discharge loss as the maximum, we cannot reduce the pitch beyond that which corresponds to this discharge loss. Furthermore, practical considerations will not permit using an excessive number of buckets. If, for instance, the buckets are bolted to the wheel disk, which is the general practice in this country, the flanges must have a certain width, thus giving a minimum pitch which may not even be as small as that resulting from the value of the discharge loss that we are willing to allow. In other words, decreasing the pitch of the buckets, for the sake of lengthening the time left for the bucket to remain in the sphere of the jet after the last particle has entered, can be carried only to a certain limit.

Beyond this limit the flow conditions of the last particle can be improved only by lengthening the arc oo_1. This we can secure by increasing the height of the arc, i. e., by making the buckets longer. But here again we are not able to go very far, as it is obvious that the buckets cannot be made excessively long in comparison with the wheel diameter. We come thus to a limit also in this direction, and if now the flow conditions are not satisfactory, the last but most effective means will have to be used, namely that of increasing the length of the arc by increasing the wheel diameter proper, or decreasing the value of K_t.

This, it is believed, shows clearly the nature of the impulse-wheel problem, when for a given size of jet the least possible wheel diameter, or the maximum value of K_t, is to be determined.

MAXIMUM TYPE CHARACTERISTIC.

In the summer of 1908 the writer had to deal with this problem quite extensively, when he was developing a type of buckets which were to be used as "standards" and which would reach the minimum value of K_t allowable for radial inward-flow turbines, with as few wheels or nozzles as possible. After all means to improve the flow conditions for the last particle, as described above, had been exhausted; that is to say, after the pitch had been reduced to a minimum and the length of the buckets had been increased to a maximum, the least wheel diameter with which a perfect reaction of the last particle could be obtained was about 10.5 times the jet diameter. With wheel diameter reduced to 9 times the jet diameter, about 2% to 3% of the jet would not react fully.

On account of the necessity of making several arbitrary assumptions in order to be able to find the position of the bucket at the time when the last particle of water has completed its flow through the bucket, the results involve some degree of uncertainty in the neighborhood of the critical point. Therefore, the values found by the writer must not be looked upon as being mathematically exact.

The question whether the wheel diameter should not be brought below $D = 10.5d$ or whether it should be allowed to go

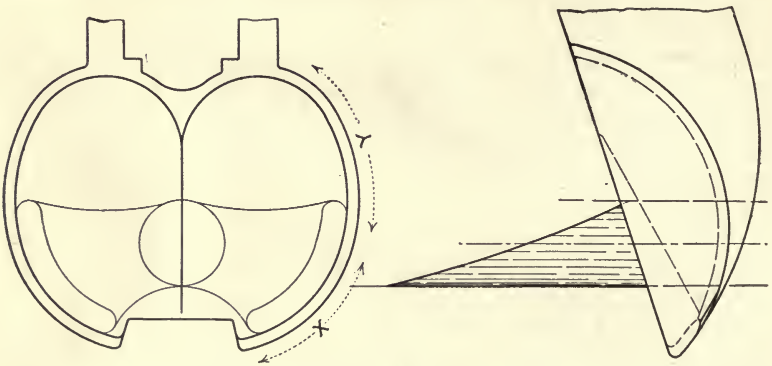

FIG. 5. PATH OF LAST PART OF JET.

somewhat lower, say, to $D = 9d$, cannot be settled by a general rule, as efficiency is not always the only deciding factor. There will be many cases in practice where the advantages of higher value of K_t, and consequently reduced number of wheels or nozzles necessary for the given power and speed, will justify a certain additional loss by imperfect reaction. A loss of 3% will often be allowable to secure these advantages. Therefore, let us establish as limiting values:

(1) $\frac{d}{D} = \frac{1}{10.5}$, making K_t about 5; for perfect reaction.

(2) $\frac{d}{D} = \frac{1}{9}$, making K_t about 6; when loss from imperfect reaction can be about 2% to 3%.

The writer does not advise trying to carry K_t higher than 6, as the loss due to imperfect reaction increases rapidly. It must be borne in mind, however, that even values of 5 or 6 can be obtained only by buckets especially adapted for such high values. These buckets must be considerably longer than standard Pelton or Doble buckets, for instance.

In these special buckets, which may appropriately be called high-speed buckets, the part X, Fig. 5, is particularly important. The smaller the wheel diameter the more inclined will be the bucket relatively to the jet at the moment when the last water particles enter, and consequently more water will be discharged at part X. This makes it necessary to design this part of the bucket with the same care as part Y, where the first and main part of the jet discharges; the bucket must be kept sufficiently deep at X and the angle β kept sufficiently small. Also another point must be mentioned in this connection: Because the increased length of the bucket makes the flow conditions at the entrance in the bucket less favorable, the practice of moving the entrance edge toward the center, as is done on all modern buckets of good design, becomes a necessity for high-speed buckets.

In the article "A Rational Method of Determining the Principal Dimensions of Water Turbine Runners," it was shown that the minimum value of K_t for radial inward-flow turbines is about 12. With favorable conditions, slightly smaller values can be reached, down to about 10. (The Allis-Chalmers Co., of Milwaukee, has built a radial inward-flow turbine for the Palmer Mountain Tunnel and Power Co. for the following data: $H = 350$, $H\text{-}P = 650$, $N = 600$; these make $K_t = 10.2$.). If now the maximum value of K_t for impulse-wheels is 6, it is apparent that we can equal the minimum limit of radial inward-flow turbines by using four single-nozzle impulse-wheels, or a four-nozzle wheel; thus the entire field of requirements for water-turbines can be covered satisfactorily by these two turbine classes alone. The conditions in this direction have been improved recently by the development of two-stage radial inward-flow turbines, a few of which have been in successful operation in Europe. These two-stage turbines extend the field of the radial inward-flow turbine in the neighborhood of the minimum limit.*

SUMMARY.

If the value of K_t computed from the given power, head and rotative speed is smaller than 12, but larger than 6, a multiple impulse-wheel must be built.

If K_t is equal to 6, a single impulse-wheel can be built, but in this case an additional loss of 2% to 3%, due to imperfect reaction of some part of the jet, must be reckoned with.

If K_t is 5 or less, a single impulse-wheel can be used without any additional loss due to imperfect reaction. However, as long as K_t is larger than about 4.2, the design must be very careful, and the buckets must be increased in length as compared with the usual standard buckets.

Best conditions for the design prevail when K_t is between 2.5 and 3.5.

After the question of the number of wheels or nozzles has been decided upon, the jet diameter will be found from formula (5).

$$d \text{ (in inches)} = 16.1 \sqrt{H\text{-}P_1}$$

where $H\text{-}P_1$ is the power of one jet per foot of head, in horsepower. The wheel diameter will be found from eq. (12).

$$D = \frac{53.67\,d}{K_t}$$

which value must check with that given by eq. (9):

$$D = \frac{72\sqrt{H}}{N}$$

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* A two-stage turbine for H, $H\text{-}P$, N requires runners of

$$K_t = \frac{N\sqrt{0.5\,H\text{-}P}}{0.5\,H\sqrt[4]{0.5\,H}} = \sqrt[4]{8}\,\frac{N\sqrt{H\text{-}P}}{H\sqrt[4]{H}}$$

Thus, if a single-stage turbine has a runner of $K_t = 12$, the runners of a two-stage turbine for the same total power, head and speed would be of $K_t = \sqrt[4]{8} \times 12 = 20.184$. Or, if we consider 12 as the minimum value of K_t for radial inward-flow *runners,* we could, in using a two-stage turbine, reach by radial inward-flow *turbines* a type characteristic as small as $12/1.682 =$ about 7.2.

NUMERICAL EXAMPLE.—Given $H = 900$ ft.; $H\text{-}P = 7{,}500$ H. P.; $N = 400$ r. p. m. Determine the arrangement and dimensions of the turbine.

$$H\text{-}P_1 = \frac{7500}{900\sqrt{900}} = 0.2778$$

$$N_1 = \frac{400}{\sqrt{900}} = 13.333$$

$$K_t = 13.333\ \sqrt{0.2778} = 7.035$$

This value is too small for a radial inward-flow turbine and too large for a single impulse-wheel. Choosing, therefore, one wheel with two nozzles, or better, two wheels with one nozzle each, K_t for each jet will be reduced to

$$K_t = \frac{7.035}{\sqrt{2}} = 4.97$$

which would require buckets of very careful design and somewhat elongated shape. On the other hand, if we decide to use four jets, i. e., either four wheels each with one nozzle, or two wheels each with two nozzles, then K_t per jet would be reduced to

$$K_t = \frac{7.035}{\sqrt{4}} = 3.5175$$

Both solutions are possible, and in deciding between them the advantages and disadvantages of each should be carefully considered. The first solution will give us a smaller and cheaper unit, but lower efficiency, whereas the second solution will give us a considerably larger unit, which will cost more and require more space in the power-house, but which will have better efficiency.

(1) The first solution will require a jet of diameter

$$d \text{ (in inches)} = 16.1\sqrt{\frac{0.2778}{2}} = 6 \text{ ins.}$$

and a wheel of diameter

$$D \text{ (in inches)} = \frac{53.67 \times 6}{4.97} = 64.79 \text{ ins.}$$

which checks with

$$D \text{ (in feet)} = \frac{72\sqrt{900}}{400} = 5.4 \text{ ft.} = 64.8 \text{ ins.}$$

(2) The second solution will require a jet of

$$d \text{ (in inches)} = 16.1 \sqrt{\frac{0.2778}{4}} = 4.24 \text{ ins.}$$

and a wheel of

$$D \text{ (in inches)} = \frac{53.67 \times 4.2432}{3.5175} = 64.8 \text{ ins.}$$

which checks with the value obtained before.

If other values for f_c, f_v and e seem to be more appropriate than 0.97, 0.47 and 0.80, respectively, the general formulas (3) and (7) should be used in computing d and D.

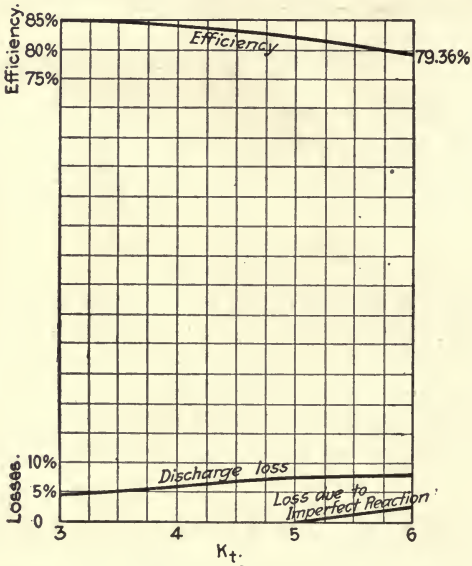

FIG. 6. PERFORMANCE OF IMPULSE WHEELS: VARIATION OF LOSSES AND EFFICIENCY WITH TYPE CHARACTERISTIC.

DISCHARGE LOSS.

At the end an interesting relation between the discharge loss, the angle β, the Type Characteristic, and the pitch or number of buckets shall be derived.

Assume that the bucket is in such a position that the center of the discharging water stream is on the impulse circle; then the peripheral velocity of this center v', is equal to the wheel velocity v.

$$v' = v = f_v \sqrt{2gH} = 0.47 \sqrt{2gH}$$

The discharge is, by Fig. 4,

$$Q = 2\,b\,w'\,(p \sin\beta - t) = \frac{\pi d^2}{4} f_c \sqrt{2gH}$$

Also,

$$w = \frac{v}{\cos\beta} = \frac{f_v \sqrt{2gH}}{\cos\beta}$$

Substituting the last value in the preceding expression,

$$\frac{2\,b f_v \sqrt{2gH}}{\cos\beta}(p \sin\beta - t) = \frac{\pi d^2}{4} f_c \sqrt{2gH}$$

Neglecting t as small compared with $p \sin\beta$, we get

$$\tan\beta = \frac{\pi d^2}{8\,b\,p}\,\frac{f_c}{f_v}$$

or, substituting for p its value $\pi D/n$ in terms of the number of buckets n,

$$\tan\beta = \frac{d^2 n f_c}{8\,b\,D f_v} = \frac{n}{8}\,\frac{d}{b}\,\frac{d}{D}\,\frac{f_c}{f_v} \qquad (13)$$

Since $\dfrac{d}{D} = \dfrac{K_t}{53.67}$, this becomes

$$\tan\beta = \frac{n K_t}{429.36}\,\frac{d}{b}\,\frac{f_c}{f_v}$$

In regard to the ratio of $\dfrac{d}{b}$, we possess no definite data, as no satisfactory experiments have been made in this respect. Turbine designers customarily assume $b = 2d$, or $\dfrac{d}{b} = \frac{1}{2}$. Putting this into our equation, and also 0.47 for f_v, and 0.97 for f_c, we obtain

$$\tan\beta = \frac{n K_t}{408} \qquad (14)$$

Thus we see that the minimum possible angle β is in direct proportion to the number of buckets used and to the value of K_t, which is a proof of the statement previously made, that for efficiency the number of buckets should be made as small as possible, because the discharge loss increases with increasing β.

The true or lateral discharge velocity of the water, c' in Fig. 4, is

$$c' = v' \tan \beta = v \tan \beta = \frac{n K_t}{429.36} \frac{d}{b} f_c \sqrt{2 g H}$$

and the discharge loss, in terms of the energy supplied, is

$$\frac{(c')^2}{2gH} = \frac{1}{2 g H} \cdot \left(\frac{n K_t}{429.36} \frac{d}{b} f_c \sqrt{2 g H} \right)^2 = \left(\frac{n K_t}{429.36} \frac{d}{b} f_c \right)^2 \quad (15)$$

For $\frac{d}{b} = \frac{1}{2}$ and $f_c = 0.97$, we obtain

$$\text{Discharge loss} = \left(\frac{n K_t}{866} \right)^2 \quad (16)$$

The discharge loss varies as the square of the Type Characteristic and as the square of the number of buckets, if the wheel is designed closely according to theory, i. e., if β is made exactly equal to the required minimum.

Numerical Example.—Take a 2½-in. jet and assume that, as K_t varies, the shape of the buckets is altered so as to get in each case the least discharge loss which can be obtained by using a certain number of buckets (the number of buckets being reduced to that required in order that loss by imperfect reaction may be zero up to $K_t = 5.0$, and 2½% at $K_t = 6$). Assume further that, at $K_t = 3$ or $D = 18d = 45$ ins., the wheel efficiency is 85%, and that the loss due to bearing friction, resistance and friction in the buckets remains unchanged. Then the following table can be figured:

The discharge loss and loss at reaction, and the total efficiency, are plotted in Fig. 6 from this table.

Wheel Diam in ins.	Ratio Wheel to jet	Type Char.	No. of Buckets	Discharge Angle		Discharge Loss		Loss due to Imperfect Reaction	Efficiency
D	D/d	K_t	n	tan β	β	Fraction of Input $(c')^2/_2g\ H$	Excess over 45″ Wheel		
45	18	2.982	20	0.1460	8° 18′	0.0453	0	0	0.85
40	16	3.355	18	0.1479	8° 25′	0.0465	0.0012	0	0.8488
35	14	3 835	17	0.1597	9° 4′	0.0541	0.0088	0	0.8412
30	12	4.475	16	0.1754	9° 57′	0.0653	0.0200	0	0.8300
27.5	11	4.88	15	0.1793	10° 10′	0.0683	0.0230	0	0.8270
25	10	5.37	14	0.1842	10° 26′	0.0720	0.0267	0.01	0.8133
22.5	9	5.97	13	0.1902	10° 46′	0.0767	0.0314	0.025	0.7936

The above-given theory may be somewhat academic, as the angle β is never made exactly equal to its minimum value but for safety is made somewhat larger (in good standard buckets, for not too high values of K_t, we find β about 10°). Also we scarcely would design a new bucket for each new value of K_t, but would use the same bucket for different wheel-diameters. Nevertheless, this theory and the curves in Fig. 6 will be reliable guides for the turbine designer both in making efficiency guarantees and in standardizing impulse-wheel buckets.